Performance Analysis of LMS & NLMS Algorithms for Noise Cancellation

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Abstract: Among various applications of adaptive filters, an important application is Interference or Noise Cancellation. The idea of adaptive interference cancellation is to obtain an estimate of the interfering signal and to remove it from the corrupted signal and hence obtain a noise free signal. In this paper performance of LMS and NLMS adaptive filter have been analyzed to remove random noise from input signal. Corrupted signal has been correlated with desired signal to cancel out noise. Both LMS & NLMS based adaptive filters have been designed & simulated using Matlab. The adapting algorithm eliminate noise from the signal by establishing correlation between noise and its estimated value. The performance of the LMS and NLMS algorithms is compared when the input of the adaptive filter is not stationary. For this, the filter uses an adaptive algorithm to modify the value of the filter coefficients, so that it acquires a recovered approximation of the signal after each iteration.

Keywords: Adaptive Filter, DSP, LMS, NLMS.

I. INTRODUCTION

In the field of signal processing interference cancellation is the technique of great importance. The technique is very much essential for speech signal transmission and processing due to the continuous progressing areas of telephone and cellular communication. Noise cancellation can be achieved by adaptive algorithms in which a filter self modifies its coefficients.

One such technique used here is Least Mean Square algorithm [Widrow and Hoff][1]. The LMS algorithm is an adequate member of the family of stochastic gradient algorithms. These kinds of algorithms use a deterministic gradient in a recursive computation of the Weiner filter for stochastic input [2]. It is not possible to exactly measure the gradient vector at each iteration even then the step size $\mu$ is suitably chosen, and the tap weight vector is computed to converge to the optimum Weiner solution. The Normalised Least Mean Square can be seen as a kind of LMS recursion that takes into consideration the variation of the signal level at the filter output.

The performance of LMS & NLMS depends upon their rate of convergence, filter coefficients and step size $\mu$[3]. It has been already researched in the past focusing on the comparison of the LMS & NLMS algorithms that, NLMS algorithm is a faster converging algorithm compared to LMS algorithm. It is concluded on the basis of convergence behaviour of two [3]. The main goal of this paper is to investigate the application of algorithms LMS & NLMS based on adaptive filtering in noise cancellation problem.

II. ADAPTIVE FILTER

A system is called adaptive when it modifies its parameters with the aid of achieving some well defined goal or target. Which depends upon the state of system and its environment. This means that the system adjusts itself such that it could respond to some phenomenon that is currently taking place in its environment[5]. Mostly the transversal structure is used in the implementation of adaptive filters defined by equation given below.

$$e(n) = d(n) - y(n) \quad (1)$$

$$y(n) = \sum_{i=0}^{N-1} w(n)x(n-i) \quad (2)$$

The Normalised Least Mean Square can be seen as a kind of LMS recursion that takes into consideration the variation of the signal level at the filter output.
III. ADAPTIVE ALGORITHMS

The LMS algorithm for a $p^{th}$ adaptive filter order & step size $\mu$ can be described by following equations [6].

**Initialization: $w(0) = 0$.**

**Computation: For $n = 0, 1, 2, ...$**

\[
X(n) = [x(n), x(n-1), ..., x(n-p+1)]^T \tag{3}
\]
\[
e(n)^2 = d(n) - y(n) \tag{4}
\]
\[
e(n)^2 = d(n) - X(n)^T w(n) \tag{5}
\]
\[
w(n+1) = w(n) + \mu e(n) X(n) \tag{6}
\]

$y(n)$ denotes the filter output, whose value is written in equation (2). $e(n)$ is estimated error signal which is obtained by substituting the value of $y(n)$ in equation (4). The computation of estimated error is based on currently estimated tap weight vector $w(n)$. Right hand side of equation (6) is tap adjustment that is applied to current estimation of $w(n)$.

The form of algorithm described by equation (2) to (6), is a complex form of LMS algorithm. At each iteration it requires knowledge of recent values of $d(n), x(n), w(n)$. The iteration is started with initial value $w(0) = 0$. The LMS is a stochastic gradient algorithm that iterates each tap weight in the filter in the direction of the gradient of the squared amplitude of an error signal with respect to that tap weight.

It is an approximation of the steepest descent gradient algorithm, that uses an instantaneous estimate of the gradient vector. The estimate of the gradient is done based on the basis of sample values of the tap input vector and an error signal. LMS algorithm iterates over each tap weight in the filter, rotating it in the direction of the approximated gradient.

In LMS algorithm a big step size is needed, to maximize the convergence speed and particularly when someone wants to address the issue of the maximum step size for stable operation of the algorithm, a theory which is valid beyond an infinitesimally small step size range is required. The results currently available for big step size use so called independence. These assumptions determine the sequence of input vector in a sequence. Although this assumption obviously violated since in the typical time series applications having N-1 elements in common, simplifies the analysis significantly. The difference between theoretical results based on this assumption and the true algorithm behaviour was investigated.

**Normalized Least Mean Square (NLMS) Algorithm:** The main drawback of the LMS algorithm is that it is sensitive to the scaling of its input $x(n)$. This makes it very hard to choose a step size $\mu$ that guarantees stability of the algorithm.

The Normalized least mean square (NLMS) is an extension of the LMS algorithm that solves this problem by normalizing with the power of the input. The NLMS algorithm can be summarized as [6].

**Initialization:** $w(0) = 0$

**Computation:** For $n = 0, 1, 2$

\[
\tilde{X}(n) = [x(n), x(n-1), ..., x(n-p+1)]^T \tag{7}
\]
\[
e(n) = d(n) - w^T(n) X(n) \tag{8}
\]
\[
\mu_{(i+1)} = \left( \mu \cdot \gamma e(n)^2 \right)^{\frac{1}{2}} \tag{9}
\]

The coefficient updating equation (9) in NLMS has been Normalized by the conjugate transpose of input vector $X(n)$. The NLMS algorithm becomes the same as the LMS algorithm except that the NLMS algorithm has a time-varying step size $(\mu)$. This step size can improve the convergence speed of the adaptive filter.

IV. SIMULATION & RESULTS

For the real time operation of this noise cancellation system, random white noise is added to the original sinusoidal input. In first simulation result original signal and estimated filter output is compared. As it requires some signal for reference for adaptive algorithm, here noise added FIR filter is taken. Simulation results shows that weighted coefficient would be more nearer in the NLMS by taking small filter size and modified step size algorithm of the filter adaptation.

In case of actual and estimated filter weight from the simulation results it can be seen that in the case of NLMS algorithm estimation is more nearer to actual values well as signal to noise ratio can be improved in case of NLMS. By applying both LMS and NLMS, comparison of the original and estimated output signals can be observed.

Figure 1 shows original periodic input that is mixed with an interfering white noise, that are uncorrelated with each other. Effect of interference is shown in figure 2. The reference input supplies a correlated version of interference for adaptive filter. By subtracting the adaptive filter output by primary input signal, noise cancellation using LMS algorithm is shown in figure 3.

Simulation results shows (figure 4) that weighted coefficient would be more nearer in the NLMS by taking small filter size and modified step-size. Results with original input and reference signal is compared to generate estimated output of NLMS.

On comparing, refer to Figure 5, it can be seen that in case of NLMS algorithm, estimation is nearer to actual value with less number of iterations.
In this paper, LMS & NLMS algorithms behaviour were studied for noise cancellation. Attempt was made to determine the effects of filter length and the step-size parameters of the two algorithms. Both LMS & NLMS algorithm based adaptive filter have been designed and simulated using Matlab. Comparisons of developed filters has shown that LMS based adaptive filter provides better MMSE and NLMS based adaptive filter provides better speed. Analysis reveals that if the LMS is chosen for de-noising, a larger step-size should be chosen, and the filter length should be kept small. For NLMS, step-size should be smaller and the filter length should also be small. But on choosing a larger step-size, the filter length should be increased to yield better performance. Moreover, NLMS provides faster Rate of Convergence, rapid tracking and low misalignment.
REFERENCES