IIR Filter Design and Analysis using Notch and Comb Filter

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ABSTRACT
Digital IIR Notch and IIR Comb filters are employed in various communication systems to eliminate unwanted narrow band interference of unwanted frequencies. In this paper the cost analysis and resource analysis of both the filters has been done. The Notch and Comb filter have been designed on Simulated MATLAB. The result shows that Comb filter provides a reduction in 83.35% of multipliers and 87.50% of adders as compared to Notch filter.

Keywords- Notch, Comb, Adders, Multipliers, Quality Factor

I. INTRODUCTION
Filters are analog circuits which performs signal processing function. It is used to specially remove unwanted frequency components from the signal to enhance wanted ones or both. Electronic filters can be:
- passive or active
- analog or digital
- High-pass, low-pass, band pass, band-reject (band reject; notch), or all-pass.
- discrete-time (sampled) or continuous-time
- linear or non-linear
- infinite impulse response (IIR type) or finite impulse response (FIR type)

A band stop filter designed to block a single frequency component is called a notch filter. In a signal processing, a comb filters adds a version of signal itself .it causes constructive and destructive interference, the frequency response of a comb filter consist of a series of regularly spaced spikes, which gives the appearance of a comb.

In signal processing, a band-stop filter or band-rejection filter is a filter that passes most frequencies unaltered, but attenuates those in a specific range to very low levels. A notch filter is a band-stop filter with a narrow stop band and high quality factor. The notch filter is to designed to attenuate a particular frequency component and has a narrow notch width. Practically, notch filter is used for eliminating the unwanted single frequency or sinusoidal interference and there are many applications of notch filter, e.g. in the field of signal processing, in electrocardiograms to removing the power line interferences, in broadcast TV to cancelling noise, etc. Narrow notch filters are used in Raman spectroscopy in optical communication, live sound reproduction in public address systems and in instrument amplifiers especially amplifiers or preamplifiers for acoustic instruments such as acoustic guitar, mandolin, bass instrument amplifier, etc. to reduce or prevent audio feedback

A comb filter is designed to block the frequencies that are integral multiples of a low frequency. A comb filter has a frequency response that is periodic function of with a period 2πL, where L is the notch frequency in the range of 0 ≤ w ≤ 2π. The comb filter is so called as because the frequency response of a comb filter consists of a series of regularly spaced spikes, giving the appearance of a comb. Comb filter is an example of filters with multiple pass bands and stop bands. Comb filter with multiple notches frequencies are used in cancellation of periodic interference. These are also used in LONAR navigation system for the suppression of cross rate interference. It is also used in digital colour television receivers for separating the luminance components from intensity information and chrominance components which contains the colour information. Cascaded Integrator-Comb filters, commonly used for anti-aliasing during interpolation and decimation operations that change the sample rate of a discrete-time system, 2D and 3D comb filters used in PAL and NTSC television systems decoder.
II. SYSTEM MODEL

The digital IIR Notch filters are generally used in because of its lower order has a narrow stop band with high quality factor. The magnitude of an ideal IIR notch filter can be represented as:

\[ H(e^{j\omega}) = \begin{cases} 0, & \omega = \omega_i \\ 1, & \omega \neq \omega_i \end{cases} \]  

(1)

Where \( i = 1 \) to \( N \), \( N \) indicates a notch frequencies. The transfer function of a single frequency notch filter having a notch at of bandwidth \( B \) can be expressed as:

\[ |H(z)| = \frac{1}{2} [1 + a(z)] \]  

(2)

Where

\[ A(z) = \frac{k_h + k_d(1 + k_h) + z^{-2}}{1 + k_d(1 + k_h)z^{-1} + k_hz^{-2}} \]  

(3)

The components \( k_d \) and \( k_h \) are given by

\[ k_d = -\cos(\omega) \]  

\[ k_h = \frac{1 - \tan(BW/2)}{1 + \tan(BW/2)} \]  

(4)

(5)

Comb filter are of two types: feedforward and feedback.

**Feedforward form:**

\[ y(n) = x(n) + \alpha x(n-M) \]  

(6)

Where ‘\( M \)’ is delay length and ‘\( \alpha \)’ is the scaling factor applied to the delayed signal. By taking Z Transform on both sides

\[ H(z) = \frac{Y(z)}{X(z)} = 1 + \alpha z^{-N} = \frac{z^M + \alpha}{z^M} \]  

(7)

Equation for frequency response of discrete-time system expressed by substituting \( z = e^{j\omega} \). Therefore the frequency response for feedforward comb filter

\[ H(e^{j\omega}) = 1 + \alpha e^{-jM\omega} \]  

(8)

By Euler’s formula, frequency response is given by:

\[ H(e^{j\omega}) = 1 + \alpha \cos(wM) - j\alpha \sin(wM) \]  

(9)

And its magnitude response is given by

\[ |H(e^{j\omega})| = \sqrt{(1 + \alpha^2) + 2\alpha \cos(wM)} \]  

(10)

Here \((1+\alpha^2)\) is a constant term where as term \(2\alpha \cos(k)\) varies periodically. Thus the magnitude response of the comb filter is periodic. The response periodically drops to a minimum value, and periodically rises to a maximum value. For positive values of \( \alpha \), the first minimum occurs at half the delay period and repeat at odd multiples of the delay frequency i.e

\[ f = \frac{1}{2M^2} \]  

(11)

The feedforward comb filter is one of the simplest finite impulse response (FIR) filters. Transfer function of the feedforward comb filter is given by:

\[ H(z) = \frac{z^M + \alpha}{z^M} \]  

(12)

**Feedback form:**

\[ y(n) = x(n) + \alpha y(n-M) \]  

(13)

General equation for feedback form is
The transfer function of feedback form is:

\[ H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - \alpha z^{-N}} = \frac{z^N}{1 - \alpha z^{-N}} \]  
(14)

and the frequency response is given by

\[ H(e^{j\omega}) = \frac{1}{1 - \alpha e^{-j\omega/2}} \]  
(15)

And the magnitude response is given by:

\[ |H(e^{j\omega})| = \frac{1}{\sqrt{(1 + \alpha^2) - 2\alpha \cos \omega}} \]  
(16)

The feedback filter has some properties common to feed forward form that is: The response periodically drops to a local minimum and rises to a local maximum. The maxima for positive values of \( \alpha \) coincide with the minima for negative values of \( \alpha \), and vice versa. For positive values of \( \alpha \), the first minimum occurs at 0 and repeats at even multiples of the delay frequency i.e.

\[ f = 0, \frac{1}{M}, \frac{2}{M}, \ldots \]  
(17)

The feed backward comb filter is a simple type of infinite impulse response (IIR) filter. If stable, the response simply consists of a repeating series of impulses decreasing in amplitude over time.

### III. SIMULATION RESULTS

Notch filters are useful to eliminate the particularly frequency component at a certain frequency. For IIR notch filter to eliminate “80” Hz interference frequency in a signal sampled at “1000”Hz filter order of “8” and quality factor of “100” is taken. The quality factor or Q-factor of the filter is a measure of how the desired frequency is isolated from other frequency for a fixed filter order; a higher Q-factor is accomplished by pushing the poles closer to the zero. Now result is determined by the use of MATLAB function.

The frequency response of a comb filter consists of regularly spaced spikes and giving the appearance of a comb. In this analysis, sampling frequency, filter order and quality factor is taken as same as the above notch filter, attenuation pass band is 1 and bandwidth(BW) is \( 2*Fs/2/8/Q \).
V. CONCLUSION

In this paper the implementation of cost for IIR notch and IIR comb filter has been analyzed. The number of adders and multipliers are 16 and 18 in IIR notch filter even the case of IIR comb it is only 2 and 3. Filter required for filter order 8, sample frequency 1000 and Quality factor 100. The IIR comb filter is much cost efficient in comparison of IIR notch filter.

<table>
<thead>
<tr>
<th>PARAMETERS</th>
<th>IIR NOTCH FILTER</th>
<th>IIR COMB FILTER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multipliers</td>
<td>18</td>
<td>3</td>
</tr>
<tr>
<td>Adders</td>
<td>16</td>
<td>2</td>
</tr>
<tr>
<td>No. of states</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Multi per input sample</td>
<td>18</td>
<td>3</td>
</tr>
<tr>
<td>Add per input sample</td>
<td>16</td>
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REFERENCE


