Swing up and Stabilization of Rotary Inverted Pendulum using TS Fuzzy

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Abstract—This paper presents an approach for the swing up and stabilization of a rotary inverted pendulum (RIP). RIP system is an unstable, multi-variable, under actuated and highly nonlinear in nature. RIP consists of a pivot arm, the pivot arm rotates in a horizontal plane by means of a servo motor. The opposite end of the arm is attached to the pendulum rod whose axis is along the radial direction of the motor. The task is to design a controller that swings up the pendulum, and keeps it in upright position. Swing up action is based on the energy principle whereas stabilization uses Takagi Sugeno Fuzzy controller. A mode controller is used to decide which control action is to be implemented. Mode controller is basically a condition check on the angle of the pendulum rod. Finally MATLAB SIMULATION results reflect the performance of the RIP system with the stated control actions.

Index Terms—Energy principle, Mode control, Rotary Inverted Pendulum (RIP), Stabilization control, Swing up control, Takagi Sugeno based Fuzzy Controller

I. INTRODUCTION

The Rotary Inverted Pendulum (RIP) system is a complex, multivariable, non-minimum, unstable, electro-mechanical system with severe nonlinearity. The control process encounters problems, such as stabilization problem, nonlinear problem and robust problem. Various control techniques can be used for the above described challenges. Conventional PID control has a simple structure, but the adjustable parameters are difficult to tune, and the overshoot of the system is likely to increase because of the systems strong instability. In addition, the conventional PID control can only monitor the control of pendulum’s angle, and can’t control displacement. On the other hand, Pole placement method has the better robustness and transient characteristics, but it significantly relies on the experience of designer to select the desired pole, so it does not have the simplicity of the PID. Whereas the T-S fuzzy controller doesn’t require an accurate mathematical model of the plant, the nonlinear system can be fuzzified into local linear model based on the empirical knowledge [1].

Gooogol Tech Rotary inverted pendulum (GRIP2001) is used in the experiment as shown in Fig. 1. It is driven by a Panasonic AC servo motor which is controlled through a motion controller card (GT400-SV).

The system consists of a pivot rod, which is free to rotate in horizontal plane. The horizontal rotation of the arm (θ) is actuated by an electromotor. The pendulum rod is connected to one end of the pivot arm and is free to rotate in vertical plane (α) and is not actuated. As the system has two degrees of freedom and only one actuator, it is called an under-actuated system [2]. First, the swing up control brings the pendulum from the downward hanging position to the upright desired position. This is achieved by repeatedly pumping energy to the pendulum rod until the pendulum is close to the upright position. Thereafter, the stabilizing control balances the pendulum in the upright position.

![Fig.1 Rotary inverted pendulum (GRIP 2001)](image-url)
II. SYSTEM MODELLING

Fig. 2 show schematic diagram of RIP system. Where ‘r’ is the length of rotating arm and ‘lp’ is the length between rotating center and mass center of pendulum rod. ‘θ’ is angle of rotating arm and horizontal axis x (clockwise is positive) and ‘α’ is angle of pendulum rod and vertical axis z (clockwise is positive).

Let ‘L’ be a Lagrange operator, x being general coordinates of system, ‘T’ is kinetic energy of system, ‘V’ is potential energy of system [2].

\[
L(x, x') = T(x, x') - V(x, x') \tag{1}
\]

![Fig. 2. Schematic diagram of RIP](image)

Generality in Lagrange equation with generalized coordinate \( \dot{\alpha} \) can be expresses as:-

\[
\dot{\alpha} = \sum_{i=1}^{n} \frac{\partial L}{\partial \dot{x_i}} - \frac{\partial L}{\partial x_i} = 0 \tag{2}
\]

Where \( l = 1, 2, 3, \ldots n \) and \( \dot{\alpha} \) is external force.

\[
T = T_{m1} + T_{m2} + T_{m3} \tag{3}
\]

Where, \( T_{m1} \) is Kinetic energy of rotating arm, \( T_{m2} \) is Kinetic energy of pendulum rod, \( T_{m3} \) is kinetic energy of mass block [2].

\[
T = \frac{1}{2}I_{m1}r^2\dot{\theta}^2 + \frac{1}{2}I_{m2}l_p^2\dot{\alpha}^2 + \frac{1}{2}m_2l_p^2\dot{\alpha}^2 + l_p^2\dot{\theta}^2 + I_p\theta^2 + m_2l_p^2\sin^2\alpha - 2m_1\dot{\alpha}\theta\cos\alpha \tag{4}
\]

Just like kinetic energy, total potential energy is also expressed as the sum of individual potential energy of the rotating arm, pendulum rod and the mass block. Total potential energy of the system can be given by:-

\[
V = m_1gr + m_2gl_p(1 + \cos \alpha) + m_3gl_p \tag{5}
\]

In orientation of generalized coordinates ‘\( \dot{\alpha} \)’ having no external force, we have

\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{\alpha_i}} - \frac{\partial L}{\partial \alpha_i} = 0 \tag{6}
\]

On expanding,

\[
\alpha'' = \frac{[m_2rl_p\theta^2 \cos \alpha + m_2l_p^2\dot{\alpha}^2 \sin^2 \alpha + m_3gl_p \sin \alpha]}{4m_2l_p^2} \tag{7}
\]
\( \alpha \) can be expressed as a function \( f(\theta, \alpha, \dot{\theta}, \alpha', \dot{\alpha'}) \).

At the balance position, i.e. the upright position these parameters will be 0. For obtaining the linear model of the system, we replace \( \sin \alpha \) with \( \alpha \) and \( \cos \alpha \) with 1 and neglect the higher order term [2]. We get,

\[
\alpha' = \frac{3g}{4l_p} \alpha + \frac{3r}{4l_p} \dot{\theta}
\]

(8)

Let the state variables of the plant be expressed as

\[
x_1 = \dot{\theta}, x_2 = \alpha, x_3 = \dot{\theta}, x_4 = \alpha, u = \theta
\]

(9)

<table>
<thead>
<tr>
<th>S.no</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( m_{1z} ), mass of rotating arm</td>
<td>0.234 Kg</td>
</tr>
<tr>
<td>2</td>
<td>( m_2 ), mass of pendulum rod</td>
<td>0.13 Kg</td>
</tr>
<tr>
<td>3</td>
<td>( m_3 ), mass of mass block</td>
<td>0.178 Kg</td>
</tr>
<tr>
<td>4</td>
<td>( r ), length of rotating arm</td>
<td>0.221 m</td>
</tr>
<tr>
<td>5</td>
<td>( l_2 ), length of pendulum rod</td>
<td>0.1975 m</td>
</tr>
</tbody>
</table>

Using these systems parameter and equation (8) and equation (9), we obtain the state space model for the system.

\[
x_1 \quad 0 \quad 0 \quad 1 \quad 0
\]
\[
x_2 \quad 0 \quad 0 \quad 0 \quad 1
\]
\[
x_3 \quad 0 \quad 0 \quad 0 \quad 0
\]
\[
x_4 \quad 0 \quad 37.2152 \quad 0 \quad 0
\]

\[
Y = \begin{bmatrix} 0 \\ \alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u
\]

III CONTROL STRATEGIES

The controlling of overall system is distributed amongst three different types of controllers performing different tasks assigned to them. Firstly, swing up controller, second is mode controller and third is stabilization controller.

A. SWING UP CONTROL

Swing up control uses energy principle, which is based on the controlling the energy of the pendulum instead of controlling its position and velocity directly. To swing the pendulum to the upright position is to give it an energy that corresponds to the upright position. This corresponds to the trajectory which passes through the unstable equilibrium at the upright position [3].

\[
E = \frac{1}{2} I(\alpha)^2 + \frac{1}{2} mgl_p (\cos \alpha - 1)
\]

(10)

Where \( J \) is the moment of inertia of pivot point

To obtain the max energy i.e. at upright position differentiate ‘E’ w.r.t. ‘t’

\[
\frac{dE}{dt} = J\alpha \alpha' - mgl_p \cos \alpha \sin \alpha = -mgl_p \alpha \cos \alpha
\]

(11)

Controllability is lost when the coefficient of \( u \) (max acceleration of pendulum rod) in the right hand side of (9) vanishes. This occurs for \( \alpha = 0 \) or \( \alpha = \pm \pi / 2 \), i.e., when the pendulum is horizontal or when it reverses its velocity. Control action is most effective when the angle \( \alpha \) is 0 or \( \pi \) and the velocity is large [3].
Fig. 3 describes the swing up controller. A low pass filter (LPF) is used to reject the change in sign due to vibration of pendulum during swing-up. A very small constant is added in velocity of pendulum to start the oscillation from rest. Value of constant decides in which direction arm rotate from rest. If value of constant is positive then arm rotate in counter clockwise and in case of negative it arm rotate in clockwise direction. In clockwise direction pendulum angle positive and in counter clockwise angle is negative continuously so slope of rate of change of angle is positive in clockwise and is negative in counter clockwise. This slope is passed through a sign block which is give either +1 or -1. \(-\pi\) to \(\pi\) block is used to keep the pendulum angle in the range of \(-\pi\) to \(\pi\). This implementation provide the max energy to the pendulum rod with consideration of the direction of the pendulum rod rotation.

\[ \text{Fig.3. Implementation of swing up control using energy principle} \]

### B. STABILIZATION CONTROLLER

Fuzzy controllers are simple, doesn’t required mathematical model of the plant, robust and effective. It consists of an three stages. First is fuzzification stage which maps the input variable to the appropriate membership function. Second stage is the rule evaluation stage which invokes the appropriate rule and produces a result for each input, then combines the results of the rules. Last is the defuzzification stage which converts the combined result back into a specific control output value [4].

TS fuzzy uses mathematical equations for the generation of the output. It expresses a non-linear system into a linear system. The output control law ‘U’ can be expressed as:-

\[ U = c_1\dot{\theta} + c_2\dot{\alpha} + c_3\alpha + c_4\dot{\alpha} + c_5 \]  

(12)

Where \(c_1, c_2, c_3, c_4, c_5\) are constants 
\(\dot{\theta}, \dot{\alpha}, \theta, \alpha\). Are the input variables to be considered. Two Gaussian type membership functions are chosen for each variable, one for negative input ‘N’ and another for positive input ‘P’. It leads to 16 rule formation, generating 16 different control signals for the control depending upon the value of inputs. Range for the position of both rotating arm and pendulum rod has been chosen as -0.3 to 0.3, whereas range for the velocity of both the arms is taken as -0.1 to 0.1. Value of constants for determination of the appropriate \(U\) is given in Table 2. [1]

Table 3 describes the formation of the rule base. Rule base is designed considering the fact that a negative input will require negative force whereas a positive input will require a positive force so as to keep the pendulum rod in upright position. Magnitude of the force depends upon the position and velocity of the input. Zero input has not been considered for the simplicity of the rule base formation [4].

### TABLE. 2. constants value

<table>
<thead>
<tr>
<th>U</th>
<th>(c_1)</th>
<th>(c_2)</th>
<th>(c_3)</th>
<th>(c_4)</th>
<th>(c_5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>U1</td>
<td>41.37</td>
<td>10.03</td>
<td>-3.16</td>
<td>-4.29</td>
<td>0.34</td>
</tr>
<tr>
<td>U2</td>
<td>40.41</td>
<td>10.05</td>
<td>-3.16</td>
<td>-4.29</td>
<td>0.24</td>
</tr>
<tr>
<td>U3</td>
<td>41.37</td>
<td>10.03</td>
<td>-3.16</td>
<td>-4.29</td>
<td>0.34</td>
</tr>
<tr>
<td>U4</td>
<td>40.41</td>
<td>10.05</td>
<td>-3.16</td>
<td>-4.29</td>
<td>0.21</td>
</tr>
<tr>
<td>U5</td>
<td>38.56</td>
<td>10.20</td>
<td>-3.16</td>
<td>-4.29</td>
<td>-0.05</td>
</tr>
<tr>
<td>U6</td>
<td>37.60</td>
<td>10.15</td>
<td>-3.16</td>
<td>-4.29</td>
<td>-0.18</td>
</tr>
</tbody>
</table>
TABLE 3. Rule base of TS fuzzy controller

<table>
<thead>
<tr>
<th>$\theta \theta$</th>
<th>$\alpha \alpha$</th>
<th>N:N</th>
<th>N:P</th>
<th>P:N</th>
<th>P:P</th>
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</thead>
<tbody>
<tr>
<td>N:N</td>
<td>U1</td>
<td>U5</td>
<td>U9</td>
<td>U13</td>
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<tr>
<td>N:P</td>
<td>U2</td>
<td>U6</td>
<td>U10</td>
<td>U14</td>
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<tr>
<td>P:N</td>
<td>U3</td>
<td>U7</td>
<td>U11</td>
<td>U15</td>
<td></td>
</tr>
<tr>
<td>P:P</td>
<td>U4</td>
<td>U8</td>
<td>U12</td>
<td>U16</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 4 gives the surface of the rule base and expounding the simplicity and linearity in the fuzzy controller used.

C. MODE CONTROLLER

The role of mode controller is to anchor which amongst the two above described control action has to be applied. It solely depends upon the position of the pendulum rod. For GRIP system when pendulum rod is in the range of ±12° from the upright position the system model is approximated as a linear system. So when pendulum is inside this range stabilization action is applies and when it lies beyond this range there appears a need of swinging up the pendulum rod.

IV. EXPERIMENTAL RESULTS

The experimental setup of Simulink model for the overall system (GRIP2001) is shown in Fig.5. Here, switch is placed to swap in between the swing up and stabilization control depending upon the position of the pendulum rod.

Googol tech uses a motion controller card (GT400-SV) which has user interfacing Modules Set Acc, Set Vel in order to set the desired velocity and acceleration of the rotating arm and Get Pos so as to have the instantaneous value of the position of the pendulum rod [2]. GT- 400-SV block is the initialization block for the motion controller card (GT400-SV).
Fig. 5 experimental setup of GRIP (2001)

Fig. 6 position of rotating arm during swing up (from initial position)

Fig. 7 pendulum rod’s angle during swing up (from upright position)

Fig. 6 and Fig. 7 shows the angular positions of rotating arm and pendulum rod. The system attains the motive of swing up in around 4.2 seconds. Fig. 8 and Fig. 9 shows the angular position of rotating arm and pendulum rod when the system is stable. System is still after 4.2 seconds and the angular position of both the rod is within ±0.5% band of steady state error.

Fig. 8 position of rotating arm in stable position (from initial position)
V. CONCLUSION

Googol Tech rotary inverted pendulum (GRIP2001) system is the plant, for which swing up is attained by energy principle within 4.2 seconds. On the other hand, stabilization is obtained using TS fuzzy controller. Fuzzy controller has an advantage over other conventional controller that it doesn’t require any mathematical model for its control action. The steady state value of the pendulum rod and the rotating arm lies within ±0.5% error band. This stabilization technique can be used for the stabilization of aircraft and stabilization of missiles in upward direction during its launch.

REFERENCES


