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Nonlinear Design for Inverted Pendulum using Backstepping Control Technique

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ABSTRACT

Backstepping control technique is a Lyapunov based nonlinear robust technique which is applicable to only strict feedback system. Backstepping technique has been applied on linear inverted pendulum which is under actuated system. Inverted pendulum is swung from its pendant position and stabilized at its upright position. Swing up is achieved by nonlinear backstepping controller but the linear inverted pendulum is having unstable zero dynamics which is stabilized linear backstepping controller. Linear backstepping controller stabilizes both the angle & position at upright position through integral regulated variable that transformed states of inverted pendulum into a global normal form on which backstepping design is applied and also with help of partial feedback linearization..

Keywords *-Nonlinear, Linear inverted pendulum, Backstepping, Zero dynamics*

1. INTRODUCTION

Inverted pendulum [1], [10] is classical example of control field in which we stabilize the pendulum at upright position from pendant position. Inverted pendulum is a highly nonlinear, under actuated and open loop unstable system .Many advanced techniques are employed such as neural, fuzzy PD, LQR, and Backstepping over conventional techniques to make system stable. In this paper, a nonlinear Backstepping [2] and Linear Backstepping are synthesized for the swing up and stabilization of inverted pendulum respectively. The backstepping is a recursive procedure in which higher order is decomposed into single order interconnected subsystem. These divided subsystems are in cascade with each other to form the overall closed loop system. The backstepping is applied on first subsystem to design a virtual control for next subsystem and design is carried out in next subsystem to design another virtual control until dynamics of subsystem contain control input explicitly.

Backstepping technique provides robustness to the system against the structured dynamics such as parameter variations and unstructured uncertainty such as un-model dynamics [6, 7]. Backstepping control technique is applicable to only strict feedback system while inverted pendulum is not strict feedback system which causes Backstepping control technique can't be applied on all states simultaneous. If backstepping control technique is applied on linear inverted pendulum there are still some states which are not controllable these states are known as zero dynamics. If these zero dynamics are stable system then system is known as minimum phase otherwise non minimum phase system.

2. MODEL OF INVERTED PENDULUM

Linear inverted pendulum is an under actuated system which means it has more number of degree of freedom than number of actuator. That is why linear inverted pendulum is not easy to get controlled. Basically linear inverted pendulum consist of pendulum of mass 'm' with rod of length 'L' which is hinged to cart pole of mass 'M' mounted on rail and moves in x direction as shown in Fig. 1

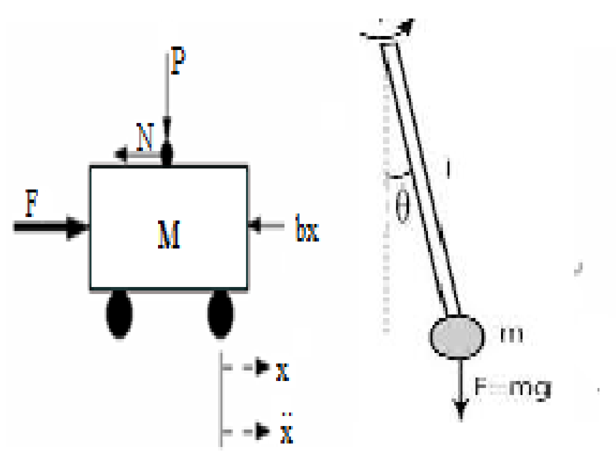

Fig. 1 Inverted Pendulum System and Free body diagram

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{M_0 \sin x_1 - m l x_2{}^2 \sin x_1 \cos x_1}{M_0 I_0 - m l \cos^2 x_1} - \frac{m l \cos x_1 u}{M_0 I_0 - m l \cos^2 x_1}$$
$$\dot{x}_3 = x_4$$
$$\dot{x}_4 = \frac{m l x_2{}^2 \sin x_1 \cos x_1 - \dot{x}_2 \cos x_1}{M_0} + \frac{u}{M_0}$$
$$y = x_1$$

where,states $\{x_1, x_2, x_3, x_4\} = \{\theta, \dot{\theta}, x, \dot{x}\}$
$M_0 = M + m$and$I_0 = I + ml^2$

3. CONTROLLER DESIGN

The Backstepping control design not only synthesized swing up controller but also stabilizes the pendulum at upright position however at stabilization point the zero dynamics is unstable. This two zone control strategy is achieved by linear controller and nonlinear controller based on Backstepping control technique.

3.1 Synthesize of Nonlinear Swing up Controller using Backstepping:

Swing up controller is used to move pendulum from pendant position to upright position which is synthesized from system (1) by backstepping control technique in which we construct a Control Lyapunov Function (CLF**)** and corresponding virtual signal is determined to stabilize the subsystem until control signal comes into dynamics explicitly. There are following steps involved into controller design.

Step 1: Stabilization of x_1 at x_{1d}
Let new variables,

$$e_1 = x_1 - x_{1d} \quad \dot{e}_1 = x_2 - \dot{x}_{1d}$$

Define CLF fore_1: Let Lyapunov function

$$V_1 = e_1{}^2/2 \qquad (2)$$ $$\dot{V}_1 = e_1 \dot{e}_1 = e_1(x_2 - \dot{x}_{1d})$$

Let $x_2 = x_{2d}$ be virtual control signal for which $\dot{V}_1$ to be negative definite $x_{2d} = \dot{x}_{1d} - k_3 e_1$ (3)

Where $k_3 > 0$ $\dot{V}_3 = -k_3 e_1{}^2 < 0$ (4)

Step 2: Stabilization of x_2 at x_{2d}
Let new variables,

$$e_2 = x_2 - x_{2d}$$

Define CLF fore_2: Let augment Lyapunov function,

$V_2 = e_1{}^2/2 + e_2{}^2/2$ (5) $\dot{V}_2 = \dot{V}_3 + e_2 \dot{e}_2$

Let $\dot{e}_2 = -k_4 e_2$ Where $k_4 > 0$ (6)

From (4), and (6),

$$\dot{V}_2 = \dot{V}_1 - k_4 e_2{}^2 < 0$$

$$\dot{e}_2 = \dot{x}_2 - \dot{x}_{2d} = -k_4 e_2 \qquad (7)$$

So from (1), and (7), Nonlinear Feedback control law for swing up,

$$u = (M_0 I_0 - (ml\cos x_1)^2)/(ml\cos x_1)\{k_4(x_2 - x_{2d}) - \dot{x}_{2d} - \frac{mglM_0 \sin x_1 + (mlx_2)^2 \sin x_1 \cos x_1}{(M_0 I_0 - (ml\cos x_1)^2}\}$$

So derivative of Lyapunov function for system (1), given by (5), is made negative definite by control law and virtual control law given by (3), and (8), which results into $e_1 = 0$ then $x_1 = x_{1d}$ and for swing up $x_{1d} = \pi$.

A. Linear Controller:

At stabilization point $\theta = \pi$, put $\ddot{\theta} = 0$ and u=0 in (1), to determine stability of zero dynamics,

$$\dot{x}_4 = (-mgl \sin x_1 - ml\cos x_1 \dot{x}_2)/ml\cos x_1 \dot{x}_2$$
$$\dot{x}_4 = g\tan x_1$$

As we know, $\sin x_1 \cong -x_1 \cos x_1$= -1 and hence,

$$\dot{x}_4 = g\, x_1 \qquad (9)$$

which is unstable zero dynamics. This zero dynamics is stabilized by linear backstepping controller. Linear backstepping controller has been synthesized with help of regulated variable by using linear model of inverted pendulum to stabilize the angle as well as position at stabilizing point. The linear model of inverted pendulum is represented around equilibrium point by equation,

$$\left.\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{M_0 \beta x_1}{(ml)^2 - M_0 I_0} + \frac{mlu}{(ml)^2 - M_0 I_0} \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{ml\beta x_1}{(ml)^2 - M_0 I_0} - \frac{I_0 u}{(ml)^2 - M_0 I_0} \end{aligned}\right\} \qquad (10)$$

where$M_0 = M + m$ and $I_0 = I + ml^2$

In addition, partial feedback linearization technique [8] is applied to reduce the complexity. Let the control law,

$u = \{M_0 - ml/I_0\}u_v + ml\beta x_1/I_0$(11)

whereu_v is the control input which convert equation into following equation,

$$\left.\begin{aligned} \dot{x}_1 &= x_2 \quad \dot{x}_2 = u_v \\ \dot{x}_3 &= x_4 \quad \dot{x}_4 = \beta x_1/I_0 - mlv/I_0 \end{aligned}\right\} \qquad (12)$$

Now backstepping control technique is applied on (10), and (12), to synthesize controller with help of regulated variable in following three steps so as to stabilize the zero dynamics.

Step 1: Introduce regulated variable q_1 and q_2

Let variable q_1 and q_2,

$$q_1 = x_1 + k_1 x_3 + k_2[mlx_4 + I_0 x_3]$$

$$q_2 = \int q_1 dt$$

$\dot{q}_1 = x_2 + k_1 x_4 + k_2 \beta x_1$

Define CLF for q_1: Let Lyapunov function,

$V_3 = \frac{1}{2} q_1^{\,2}$ then $\dot{V}_3 = q_1 \dot{q}_1$ (13)

q_{2d} is the virtual control for which $\dot{V}_3$ will become negative definite then,

$q_{1d} = -k_1 x_4 - k_2 \beta x_1 - c_1 q_1 - c_2 q_2$ (14)

Where c_1 and $c_3 > 0$.

$\dot{q}_1 = x_2 - q_{1d} - c_1 q_1 - c_2 q_2$

Let $q_3 = x_2 - q_{2d}$

$\dot{q}_1 = q_3 - c_1 q_1 - c_2 q_2$ (15)

Step 2: Stabilize regulated variable q_2

Define CLF for q_2: Let augment Lyapunov function,

$$\left.\begin{aligned} V_4 &= q_1^{\,2}/2 + c_2 q_2^{\,2}/2 \\ \dot{V}_4 &= q_1 \dot{q}_1 + c_2 q_2 q_1 \end{aligned}\right\} \quad (16)$$

From (15), and (16),

$\dot{V}_4 = -c_1 q_1^{\,2} + q_1 q_3$

Step: 3 Stabilize regulated variable q_3,

$$q_3 = x_2 - q_{1d}$$

$$\dot{q}_3 = u_v + k_1[\beta x_1/I_0 - mlv/I_0] + k_2 \beta x_2 + c_1[q_3 - c_1 q_1 - c_2 q_2] - c_2 q_1$$

Define CLF for q_3: Let augment Lyapunov function,

$$V_5 = V_4 + \frac{1}{2} q_3^{\,2} \quad (17)$$

Now, for $\dot{V}_5$ to be negative definite,

$$u_v = d_3[d_1 q_1 + d_2 q_3 + k_1 \beta x_1/I_0 + k_2 \beta x_2 - c_1 c_2 q_2] \quad (18)$$

where,

$$d_1 = \left(1 + c_2 - c_1^2\right) d_2 = (c_1 + c_3)$$

$d_3 = I_0/I_0 - k_1 (ml)^2$

$\dot{V}_5 = -c_1 q_1^{\,2} - c_3 q_3^{\,2}$

Where $c_1, c_2, c_3 > 0$

So linear controller for stabilization from (11),

$$u = d_3 d_4 \{d_1 q_1 + d_2 q_3 + k_1 \beta x_1/I_0 + k_2 \beta x_2 - \dot{c}_1 c_3 q_2\} \quad (19)$$

where, $d_4 = M_0 - ml/I_0$

So derivative of lyapunov function for system (10), given by (17), is made negative derivatives by control law and virtual control law given by (14), and (19).

4. SIMULATION

Simulation results of real time implementation are shown in figure with initial condition $x_1(0) = 0$ and $x_3(0) = 0$. System parameter and designed parameters are as shown in Table1 and Table 2 as shown below

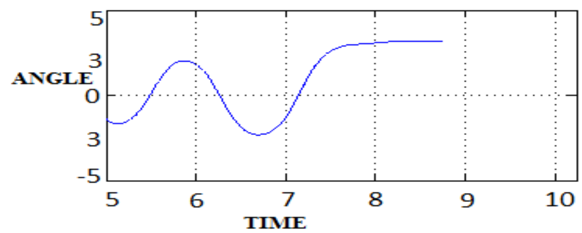

Fig 2 Swing up of inverted pendulum

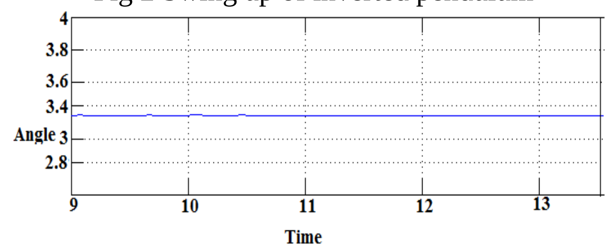

Fig 3 Stabilization of pendulum through integral regulated variable

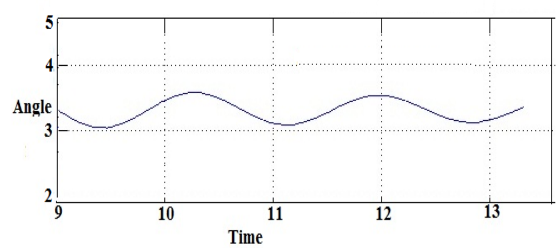

Fig 4 Stabilization of inverted pendulum using regulated Variable

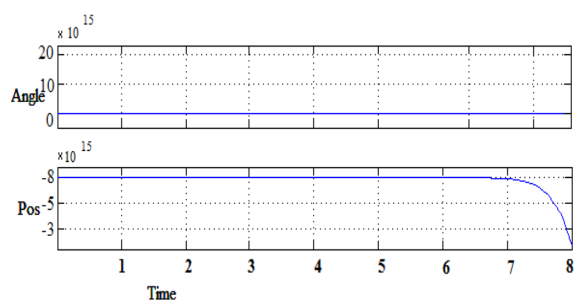

Fig 5 Unstable zero dynamics

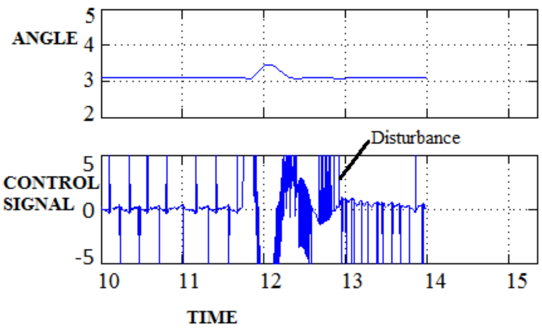

Fig 6Accommodation of disturbance by controller

TABLE I. System parameters

M	m	L	I	g
1.096	.109	.25	.0034	9.8

TABLE II. Designed Parameters

k_1	k_2	k_3	k_4	c_1	c_2
.3	.2	25	44	28	35

The inverted pendulum is swung up by nonlinear backstepping controller as shown in Fig. 2 within 8sec without considering friction which causes it more time to swing up. The stabilization of inverted pendulum at upright position is controlled by linear backstepping control using regulated variable and integral regulated variable as shown in Fig. 3 and Fig. 4 which shows that integral regulated variable minimizesthe steady.

At stabilization, zero dynamics are represented by,

$$\begin{bmatrix}\dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}0 & 1\\ -p1 & -p2\end{bmatrix}\begin{bmatrix}x_3\\ x_4\end{bmatrix}$$

where,

$$p1 = d_3(d_1 + d_2c_1)k_1$$

and

$$p2 = d_3[(d_1k_2 + d_2k_1 + d_2c_1k_2ml)]$$

So for zero dynamics to be stable,

$$d_3 > 0,\ 0 < k_1 < \frac{I_0}{(ml)^2} = 2.6$$

If $k_1 = 2.8$ the system becomes unstable which is shown in Fig. 5.

Controller able to accommodate the disturbance and stabilize the pendulum at equilibrium point which is shown in Fig. 6 when disturbance is introduced in system at t= 12.5 sec which shows robustness of controller for bounded disturbance

5. CONCLUSION

Inverted pendulum is controlled by two zone control strategy known as Swing up controller and stabilization controller. Inverted Pendulum which have unstable zero dynamics at upright has been stabilized by linear controller using integral regulated variables while swing up controller is synthesized by nonlinear backstepping control technique. From simulation results it is evidentthat Backstepping control technique is very robust irrespective of parametric or un-modeled dynamics. Analytical complexity is increased as the order of system is increased and also saturation problem is occurred due to derivative of virtual control.

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