STABILITY ANALYSIS OF MODEL PREDICTIVE CONTROLLER WITH CONSTRAINTS VARIATION FOR MAGNETIC LEVITATION SYSTEM

Shailendra Chauhan¹, Himansh Kumar², Varunendra Kumar Singh³, Ashis Behera⁴
¹Department of Electronics & Communication, IIT Roorkee
²Instrument Design Development Center, IIT Delhi
³, ⁴Department of Electronics & Communication, IIT Roorkee

ABSTRACT
In this paper stability of Model Predictive Controller has been analyzed under various constraints on inputs and outputs. The state estimation has been used to ensure the stability of the system. Since Model Predictive Controller is based on prediction theory in which input signal is computed at each sampling interval and output is estimated, so computed control signal at each sampling time is optimized over a specified horizon. The first calculated value of input signal is implemented and rests are discarded. Then this process is repeated for the next step. The conventional controller like Fuzzy PID [11][12] gets failed to control the system effectively under such variation on constrained, so an accurate model and complex algorithm are required to design a self-tuned Fuzzy PID controller to accommodate the ability to handle the constrained variation.

Keywords—Model predictive controller, receding horizon, magnetic levitation system, state estimation, stability

I. INTRODUCTION
Model predictive control (MPC) is a technique in which control action is calculated and fed to the system before any change in the output value actually occurs. The basic approach of this technique is to solve an optimization problem for a finite future at current time and to implement the first optimal control input as the current control input. In this technique primarily the performance index is optimized over a finite future horizon at present time and then first optimal control input is implemented as the current control input. The behavior of the system is analyzed over a specified horizon and manipulated variable is estimated using the mathematical expression of the model. The step size of manipulated variables i.e. output of the system is chosen in such way that the estimated output remain under desirable limits. Since practical problems are dominated by constraints and it can be categorized widely into two sections-input constraints and output constraints[6][9]. The input constraints are imposed by physical limitation of the actuator which can not be violet under any condition. In similar fashion it is also required to keep the output within a certain limits. For the magnetic levitation system, assuming a feasible set of inputs and outputs constraints various results on global stability have been shown using a specified prediction and control horizon.

II. MAGNETIC LEVITATION SYSTEM MODELING
Magnetic levitation refers to suspending an object in air without any physical support just using magnetic force to counter balance weight of the object and any other force acting on the object. If magnetic attraction is used to control the ball in air then this phenomena is called magnetic levitation. Control action is analyzed over a specified horizon and manipulated variable is estimated using the mathematical expression of the model. In the past, magnetic levitation was attempted by using permanent magnets; however it is feasible to attain a stable equilibrium using electromagnet magnets. [2] Thus different techniques are used to levitate an object specially using electromagnet, and controlling the current flowing in the electromagnet via a feedback loop to stabilize the system. Electromagnetic suspension works via the force of attraction between an electromagnet and the object. If the object gets too close to the electromagnet, the current in the electromagnet must be reduced. If the object gets too far, the current in the electromagnet must be increased to pull back the object to the equilibrium point. Electromagnetic levitation works via electromagnetic force of repulsion and we use feedback in the system in order to levitate the object. The dynamics of the coil are usually represented as an equivalent R-L circuit in series. When the electric circuit of the coil is driven by a voltage source; Kirchhoff’s voltage law gives the relationship [2]...
\[ v = \frac{dq}{dt} + R i \]  

(1)

Where \( v \) = the source voltage  
\( R \) = series resistance of the circuit  
\( \Phi = L(x) \) = the magnetic flux linkage

The forces acting on the suspended object are its weight downwards, electromagnetic force upwards and viscous friction opposite to the velocity in vertical plane. Thus its dynamics in vertical plane can be described as:

\[ m \frac{d^2x}{dt^2} = mg - k \dot{x} - F(x, i) \]  

(2)

Where \( m \) = mass of the suspended object  
\( g \) = acceleration due to gravity  
\( x \) = distance of ball surface from the bottom of the electromagnet (\( x \geq 0 \))  
\( k \) = coefficient of viscous friction  
\( i \) = current in the coil  

\( F(x, i) \) = force generated by the electromagnet

Now energy stored in the electromagnet is given as

\[ E(x, i) = \frac{1}{2} L(x) i^2 \]  

Thus the force \( F(x, i) \) is calculated using (3) as

\[ F(x, i) = -\frac{\partial E}{\partial x} = \frac{L_a a}{2} \frac{i^2}{(a + x)^2} \]  

(4)

Using above expression of \( F(x, i) \) in (2) the differential equation governing the motion of the suspended object is given as

\[ m \frac{d^2x}{dt^2} = g - \frac{k}{m} \dot{x} - \frac{L_a a}{2m} \frac{i^2}{(a + x)^2} \]  

(5)

Equation (4) and (5) govern the dynamics of the whole plant including the electromagnet and the suspended object.

Using the data provided in table 1[2] Transfer function of the Magnetic Levitation System can be given as

\[ G_{o(s)} = \frac{77.8421}{0.03115s^3 - 30.5250s} \]  

(6)

Equation 6 can also be presented below in state space form also

\[ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 2499.1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} U_{in} \]  

(7)

### Table 1. Parameters for Magnetic Levitation System

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Suspended Mass</td>
<td></td>
<td>0.1</td>
</tr>
<tr>
<td>Coil Inductance</td>
<td></td>
<td>0.02</td>
</tr>
<tr>
<td>Incremental Inductance</td>
<td></td>
<td>0.01</td>
</tr>
<tr>
<td>Coil Resistance</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Gravity Acceleration</td>
<td></td>
<td>9.81</td>
</tr>
<tr>
<td>Coefficient of Viscosity</td>
<td></td>
<td>0.001</td>
</tr>
<tr>
<td>Constant ( \frac{1}{\mu} )</td>
<td></td>
<td>0.05</td>
</tr>
</tbody>
</table>

### III. MODEL PREDICTIVE CONTROL STRATEGY

The mathematical analysis of the system and its derivation of the relations between system inputs and outputs provide the general model which would be valid for a whole range of system's inputs and states. On the other hand, there are usually a lot of unknown constants and relations when performing the mathematical analysis. Therefore, modelling by mathematical analysis is suitable for simple controlled systems with small number of parameters or for obtaining basic information about the system. The basic functional building block has been shown in Fig. 1 in which reference input signal is in continuous time where as controller and plant outs are in discrete time [10].

\[ X(k + 1) = Ad X(k) + Bd U(k) \]  

(8)

\[ Y(k) = Cd X(k) \]  

(9)

\[ Ad = \begin{bmatrix} 0 & 1 \\ 2499.1 & 0 \end{bmatrix} \]

\[ Bd = \begin{bmatrix} 0 \\ 2499.1 \end{bmatrix} \]

\[ Cd = \begin{bmatrix} 0 & 2499.1 \end{bmatrix} \]

\[ Dd = [0 \ 0] \]

\[ X(k + 1) = Ad X(k) + Bd [U(k - 1) + \Delta U(k)] \]  

(10)

\[ X(k + 1) = [AdBd] \begin{bmatrix} X(k) \\ U(k-1) \end{bmatrix} + Bd \Delta U(k) \]  

(11)
Let assume \( \Psi(k) = \begin{bmatrix} X(k) \\ U(k-1) \end{bmatrix} \) (12) 
\[
\Psi(k+1) = \begin{bmatrix} X(k+1) \\ U(k) \end{bmatrix} = \begin{bmatrix} Ax(k) + Bu(k-1) + DU(k) \end{bmatrix} 
\]
(13) 
\[
\Psi(k+1) = \begin{bmatrix} AdX(k) + Bdu(U(k-1) + DU(k)) \end{bmatrix} U(k) 
\]
(14) 
\[
\Psi(k+1) = [Cd 0] \begin{bmatrix} X(k) \\ U(k-1) \end{bmatrix} 
\]
(15) 
\[
\Psi(k+1) = A\Psi(k) + BDU(k) 
\]
(16) 
\[
Y(k) = C \Psi(k) 
\]
(17) 
\[
Y(k) = Cx(k) 
\]
(18) 

3.1 Prediction control law

Let predicted output of the controller represented as [10].
\[
\hat{y}(k) = PDU(k) + Q\Psi(k) 
\]
(19) 
\[
\hat{y}(k) = \begin{bmatrix} \hat{y}(k+1) \\ \hat{y}(k+2) \\ \vdots \\ \hat{y}(k+N) \end{bmatrix} 
\]
(20) 
\[
Q = \begin{bmatrix} CA & CA^2 & \cdots & CA^{N-1} \\ 0 & CA & \cdots & CA^{N-2} \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & CA \end{bmatrix} 
\]
(21) 
\[
P = \begin{bmatrix} CB & \cdots & 0 & \cdots & 0 \\ C\hat{X}(k) & CB & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ CA^{N-2}B & CA^{N-3}B & \cdots & CA^{N-k}B \end{bmatrix} 
\]
(22) 

The performance index can be given as
\[
J(DU(k)) = \sum_{i=1}^{N}(y(k+i) - \hat{y}(k+i))^2 + \rho \sum_{i=1}^{M}(DU(k+i-1))^2 
\]
(23) 
\[
J(DU(k)) = (R - \hat{y})(R - \hat{y}) + \rho DU^T DU 
\]
(24) 
\[
J(DU(k)) = (R - PDU - Q\Psi(k))^T(R - PDU - Q\Psi(k)) + \rho DU^T DU 
\]
(25) 
\[
J(DU(k)) = \rho DU^T DU 
\]
(26) 
\[
J(DU(k)) = 2 \rho DU^T DU + 2 DU^T DU 
\]
(27) 
\[
J(DU(k)) = 2 DU^T DU 
\]
(28) 
\[
J(DU(k)) = 2 DU^T DU 
\]
(29) 
\[
J(DU(k)) = 2 DU^T DU 
\]
(30) 

3.2 Stability of MPC under with constraints using state estimation

Magnetic Levitation System can be presented in state space form as [6]
\[
X(k+1) = AX(k) + BU(k) 
\]
(26) 
\[
Y(k) = CX(k) 
\]
(27) 

The current control \( \hat{x}(k/k) \) system model can be calculated as follows
\[
\hat{x}(k/k) = A \hat{x}(k|k-1) + BU(k-1) 
\]
(28) 
\[
\hat{x}(k/k) = \begin{bmatrix} A \hat{x}(k|k-1) & KeY(k) - C \hat{x}(k|k-1) \end{bmatrix} 
\]
(29) 
\[
\hat{x}(k/k) = \begin{bmatrix} A \hat{x}(k|k-1) & KeY(k) - C \hat{x}(k|k-1) \end{bmatrix} 
\]
(30) 
\[
\hat{x}(k/k) = \begin{bmatrix} A \hat{x}(k|k-1) & KeY(k) - C \hat{x}(k|k-1) \end{bmatrix} 
\]

Predicted future state
\[
\hat{x}(k+i|k) = A \hat{x}(k+i-1|k) + BU(k+i-1) 
\]
(31) 

So state error can be estimated as follows
\[ X_p(k|k) = X(k) - A \hat{X}(k|k) \quad (31) \]
\[ = (A - KeCAx(k - 1)k - I) \quad (32) \]

So presently predicted state
\[ \hat{X}(k|k) = (A - KeCA)\hat{X}(k - 1)k - 1 \]
\[ + (B - KeCB)U(k \sim 1 + Ke(Y(k)\quad (33) \]

The predicted state can be represented in the terms of control signal and manipulated variable using appropriate gain as shown in Fig 2. The appropriate combination of parameters ensures the stability of the system.

IV. SELECTION OF PARAMETERS AND SIMULATION

The design parameters of Model Predictive Controller are given as in Table 1 in which \( N \) is prediction horizon over which prediction is done and \( M \) is no of control horizon over which optimization is done. \( \omega_1 \) & \( \omega_2 \) are input output weight factor respectively.

<table>
<thead>
<tr>
<th>Controller Design Parameters</th>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design Parameters</td>
<td>( t )</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>( N )</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>( M )</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>( \omega_1 )</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>( \omega_2 )</td>
<td>0.1</td>
</tr>
</tbody>
</table>

The simulation of Model Predictive Controller for Magnetic Levitation System is show in Fig.3.

V. RESULTS AND CONCLUSION

In this paper it has been explored the potential benefits of Model Predictive Controller using state estimation technique. This controller, with the given parameter as shown in Fig 3 is analyzed under variation of input/output constraints. As the constraints are relaxed it is easy for the controller to control the system because it would generate the required control signal.

Fig. 4 shows the response of the system when manipulated variable lie between -1.5 and 1 and Fig. 5 shows the response when constraint is got harden i.e. manipulated variable gets limited between -1.5 and 0. Fig. 6 shows the response when manipulated variable lie between -1 and 1. Fig. 7 shows the response when manipulated variable lie between -1.5 and -1, so under this high restriction on manipulated variable it became unstable henceforth it can be concluded that even putting a very strong constrained on manipulated variable of the predictive controller, system response is not getting affected appreciably.
Fig. 5 Response of system when plant output lie between -2 and 2 and controller output lie between -1.5 to 0

Fig. 6 Response of system when plant output lie between -2 and 2 and controller output lie from -1 to 1

Fig. 7 Response of system when controller output lie between -1.5 to -1

The Fig. 8 shows the system response when constraints are imposed upon the controller output and plant output both -5 to 5 and -5 to 5 respectively and now if controller output gets changed between -2 to 2 as shown in Fig 9, response is under desirable limit. Again hardening the controller output constraints between -1 and 1 a single undershoot gets increased how ever system gets stabilized quickly as shown in Fig. 10. In case if we put the stringent constraints such as controller output (.5 to .5) and plant output (-1 to 1) as shown in Fig 11, system gets destabilized.

Fig. 8 Response of system when plant output lie between -.5 to 5 and controller output lie from -5 to 5

Fig. 9 Response of system when plant output lie from -.5 to 5 and controller output lie from -2 to 2

Fig. 10 Response of system when plant output lie from -2 to 2 and controller output lie from -1 to 1
REFERENCES


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