Implementation of LQR Mapped Sensory Fuzzed Fuzzy Logic Controller on Inverted Pendulum System

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ABSTRACT
When designing a controller, the very first step is mathematical modeling of the concerned plant. As we know that during this mathematical modeling several mathematical approximation, uncertainties in the form of system parameter variations, plant nonlinearities and disturbances are involved. Although the conventional PID controller is vastly used controller but it is linear in nature thus it demands a very accurate mathematical model of the plant. Suppose for the given operating conditions, the system has nonlinear behavior then the conventional PID controller may perform unsatisfactorily hence in such conditions fuzzy controller may perform better. As fuzzy controller can be designed with nonlinear characteristics thus a well-designed fuzzy controller can handle plant uncertainties and inherent nonlinearities in quite effective manner. But if the number of inputs is more then while designing the rules for the fuzzy controller will be large and complex. Having large number of rules may degrade the performance of the controller and sometimes cause the rule explosion in FLC. To overcome this problem LQR (Linear Quadratic Regulator) fusion technique is proposed which reduces the possibility of rule explosion by reducing the number of inputs as well as the number of rules. As in case of Inverted pendulum system, there are four inputs to conventional fuzzy controller which has been reduced to two inputs with LQR fusion technique. The performance of the designed controller is evaluated in simulations as well as in real time using MATLAB and SIMULINK on Inverted Pendulum. The results shown in this report show the satisfactory performance of the controller in each case.

Keywords— Fuzzy logic controller, LQR fusion, Inverted pendulum system.

I. INTRODUCTION
Normally plants can be controlled by conventional controllers. Over 90% industries are using conventional PID controller for their proper operation due to its simplicity. Just three gains are sufficient to control the plant. But when one talks about the research, the fuzzy logic is new and widely researched. Fuzzy logic controllers are also termed as an intelligent controller that’s why it finds its application in intelligent devices.

Many researchers suggested that fuzzy logic controllers are good over conventional controllers. In research work fuzzy logic controller is more efficient. The enactment of fuzzy control is an replicated of the control laws that humans use. Creating machines to imitate human skill in control provides us an opportunity to design controllers for complex plants whose mathematical models are not easy to identify. Fuzzy logic controllers serve the same function as most conventional controllers, but they manage complex control problems through heuristics. The mathematical model required for the fuzzy logic controller just in the form of transfer function or sate space form. Fuzzy logic controller itself don’t use any form of mathematical equations. It just takes input and provides control signal for the plant. The application of fuzzy control to large-scale complex systems is not, by no means, trouble-free. For such systems the number of the fuzzy IF-THEN rules as the number of sensory variables increases very quickly to an unmanageable level. When we take into account more input variables in control system, the number of rules grows exponentially, if we have \( l \) possible values for each of \( n \) variables, we must describe control corresponding to all \( l^n \) possible combinations of input values. Here the method of sensory fusion is studied in an attempt to reduce the size of the inference engine for large-scale systems. This structure reduces the number of rules considerably [2].But the adequate parameters should be estimated for the implementation of this technique. Much reliance has to be put on the experience of the operator with respect to the find these parameters[3]. In this work we will find the estimation of the parameters of the sensory fusion method using LQR Mapping Based Information Fusion. It is an appropriate technique to find the parameters in a large search space. Also in the optimization problems it has shown efficient and reliable results [4][6].

II. LQR MAPPING BASED INFORMATION FUSION
This method uses the characteristics of LQR which belongs to modern control theory into fuzzy logic. Fuzzy controller may also be designed for the even for the complex system through the reduction of the variables. Inverted pendulum system is a multi-sensor system; multi-sensor is the basis of information fusion, and
multi-source information is its object. The information fusion is that the multiple sensor or multi-source information is treated comprehensively, in order to obtain more accurate and reliable conclusion [6]. The approximate linear state equation near the natural upward equilibrium position of the inverted pendulum system can be obtained after some assumptions and approximations. It is well known that the linear system has characteristics of direct integration, so the inverted system state variables can be changed into integrated error E and error change EC by constructing a linear fusion function.

In this paper, A linear fusion function is obtained through the LQR gain mapping technique on inverted pendulum system. The state space equation 6 of inverted pendulum is used for the calculation of the linear fused function.

Fusion function design steps [3] combining with optimal control are given as follow:

- Choose the quadratic objective function as

\[ J = \frac{1}{2} \int_{0}^{\infty} (XQX + U'R^2U)dt \]  

For the inverted pendulum system, weighting matrix Q and R are used to balance the weight of the system’s state vector X and U. Because of Q being a semi-definite matrix and R being a definite matrix, the objective function is non-negative. On output, disturbances affecting the system, give an appropriate U that is called optimal control to make the system return to equilibrium position as soon as possible and at the same time make the objective function minimum.

- Calculate the state feedback matrix K that can make the inverted pendulum system basically stable through LQR theory. For, R=1 and Q=[1000 0 200 0], and the value of K is

\[ K = [-31.623 -20.151 72.718 13.155] \]  

- Formula of the fusion function \( F_1(X) \) is given by (3).

\[ F_1(X) = \frac{1}{||K||} \begin{bmatrix} K_X & K_\theta \\ 0 & 0 & K_X & K_\theta \end{bmatrix} \]  

This formula can be obtained with the help of state feedback matrix K. Where ||K|| is given by (4).

\[ ||K|| = \sqrt{(K_X)^2 + (K_\theta)^2 + (K_X)^2 + (K_\theta)^2} \]  

Fusion function with its numerical value has been shown in (5) [3][4].

\[ F_1(X) = \begin{bmatrix} -0.38164 & 0.8776 & 0 & 0 \\ 0 & 0 & -0.2432 & -0.15875 \end{bmatrix} \]  

With the proper choice of the PD gains, the linear PD controller can easily be made to have the same shape as the fuzzy controllers like convention controller. This type of control surface is representing the control surface for linear FLC controller. So nonlinearity handling capability is not good enough. The linear relationship between input/output can be seen through the Fig.2.3.

Linear FLC controller uses three membership function Negative (N), Zero (Z), Positive (P) and have two inputs error (E) and change in error (EC). So there are total 9 rules that are sufficient to control the IP.

### III. CONTROL SCHEME

\[ a_1 = -0.38164, a_2 = 0.8776, a_3 = -0.2432 \text{ and } a_4 = -0.15875 \]
Proposed method earlier has been applied on inverted pendulum for the verification of the results with designed controller using proposed techniques. Inverted pendulum is highly unstable and nonlinear in nature therefore to stabilize the pendulum in upright vertical position is a challenging task.

\[
\begin{bmatrix}
\dot{x} \\
\dot{\phi} \\
\phi \\
\dot{\phi}
\end{bmatrix} = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & \frac{3g}{4l} & 0
\end{bmatrix} \begin{bmatrix}
x \\
\dot{x} \\
\phi \\
\dot{\phi}
\end{bmatrix} + \begin{bmatrix}
0 \\
1 \\
0 \\
\frac{3}{4l}
\end{bmatrix} u' \tag{6}
\]

\[
y = \begin{bmatrix}
x \\
\phi \\
\dot{x} \\
\dot{\phi}
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
x \\
\dot{x} \\
\phi \\
\dot{\phi}
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix} u' \tag{6}
\]

**IV. SIMULATION AND RESULTS**

Simulations are performed on the state equations of the IP. All the states position, angle and angular velocity are converging at the equilibrium point that is zero as shown in Fig. 5.

Fig. 6 shows the card position of inverted pendulum in transient as well as in steady state. From Fig. 6, one can depict that the settling time of the cart is around 2 sec with negligible overshoot which is acceptable in real time also.

Fig. 6 shows the stabilization of the pendulum at inverted position. The settling time of the pendulum is less than 2 sec with negligible peak overshoot of the order of $10^{-3}$ and zero steady state error. With a large overshoot present in the system, it may be very difficult for the controller to stabilize the pendulum at desired position and may cause actuator breakdown. The performance of the overall closed loop system when subjected to random white noise is shown in Fig. 6. It shows the good noise handling capability of the Fuzzy controller. The steady state error of the pendulum as well as of the cart lies well within 2% band which is acceptable for the good real time performance of the system.

The state equations used in simulation are,
VI. CONCLUSION
For the tuning of the scaling blocks LQR mapping based information fusion method has been studied and implemented in this paper. The fusion of the input variables of the fuzzy controller makes it possible to reduce significantly the dimensions of the control problem. In our approach the problem of arbitrary search for the required parameters was replaced with LQR mapping based information fusion. The designed controllers are implemented on inverted control system. The point of attraction of this fusion method is that, four states of the inverted pendulum is controlled by just four rules of the fuzzy controller and the results, carried out in MATLAB and presented in the paper verify the satisfactory performance of the designed controller. Simulation results show that fuzzy control method of inverted pendulum system based on information fusion has good response characteristics to verify the feasibility of the proposed method.

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