Natural Frequency based Swing up of Rotary Inverted Pendulum and its Stabilization

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ABSTRACT
This paper aims at the applications of natural frequency of inverted pendulum and Takagi-Sugeno (TS) fuzzy control in the swing up and stabilization of Rotary inverted pendulum (RIP) respectively. RIP is one of the important multivariable system in control to test the performance of any controller due to its nonlinear, unstable, non-minimum phase and under actuated nature. Many techniques based on state variable of pendulum are used for swing up of pendulum from down hanging position to vertical upright position. But a more simple way to swing up is based on natural frequency of pendulum. It can be calculated either experimentally or based on equation of time period of pendulum. During swing up pendulum change its direction at every half of time period of pendulum except very first time where it changes its direction at one fourth of its time period. If we apply constant control input to the arm of pendulum based on this time period, we can swing up the pendulum without any feedback for RIP system. The advantage of fuzzy logic controller is its aptitude to deal with nonlinearities, uncertainties without accurate mathematical model of the system. TS fuzzy controllers are close to gain scheduling approaches. Several linear controllers are defined, each valid in one particular region of the controller's input space. In between the swing up and stabilization a mode controller is used which is basically a condition check on the angle of the pendulum rod. Finally Simulink based experimental results are used to verify and validate the performance of the above controllers on RIP system.

Keywords – Natural frequency based swing up controller, TS Fuzzy controller, RIP

I. INTRODUCTION
The Rotary Inverted Pendulum (RIP) system is a multivariable, unstable, complex, non-minimum, unstable system. While controlling a plant various problems like stabilization problem, nonlinear problem and robust problem has to be considered for better results. Various control techniques can be used for the above described challenges. Conventional PID control has a simple structure, but the adjustable parameters are difficult to tune, and the overshoot of the system is likely to increase because of the systems strong instability. In addition, the conventional PID control can only monitor the control of pendulum’s angle, and can’t control displacement. On the other hand, Pole placement method has the better robustness and transient characteristics, but it significantly relies on the experience of designer to select the desired pole, so it does not have the simplicity of the PID. Whereas the T-S fuzzy controller doesn’t require an accurate mathematical model of the plant, the nonlinear system can be fuzzified into local linear model based on the empirical knowledge [1].

Various techniques are described for swinging up of pendulum rod that is to bring the pendulum rod from down hanging stable position to the upright unstable position. But these techniques use feedback mechanism. Swing up becomes bit difficult and tricky if there appears error in the feedback mechanism may be due the improper working of sensors. Therefore, a more simple way to swing up is based on natural frequency of pendulum. It can be calculated either experimentally by measuring time period using stopwatch or by calculating the time period of the oscillation based on the pendulum rod. During swing up pendulum change its direction at every half of time period of pendulum except very first time where it changes its direction at one fourth of its time period. If we apply a changing constant control input based on time period to the arm of pendulum, we can swing up the pendulum without any feedback form RIP system.

II. MATHEMATICAL MODEL OF THE SYSTEM
GoogolTech Rotary inverted pendulum (GRIP2001) is used in the experiment as shown in Fig. 1. It is driven by a Panasonic AC servo motor which is controlled through a motion controller card (GT400-SV). The system consists of a pivot rod, which is free to rotate in horizontal plane. The horizontal rotation of the arm (θ) is actuated by an electromotor. The pendulum rod is connected to one end of the pivot arm and is free to rotate in vertical plane (α) and is not actuated. As the system has two degrees of freedom and only one actuator, it is called an under-actuated system [2].
Taking arm acceleration $\ddot{\alpha}$ as input we get the following differential equation for RIP

$$\frac{4}{3} m_p l_p^2 \ddot{\alpha} - m_p r l_p \ddot{\theta} - m_p g l_p \alpha = 0$$  \hspace{1cm} (1)

After simple algebraic manipulation in Eq. (2.24) we obtain the following linear, state-space representation [5] of the RIP-system for GRIP (2001)

$$\begin{bmatrix} \dot{\theta} \\ \dot{\alpha} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \alpha \\ \alpha \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.84 \end{bmatrix} \ddot{\alpha}$$  \hspace{1cm} (2)

$$y = \begin{bmatrix} \theta \\ \alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \alpha \\ \alpha \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \ddot{\alpha}$$  \hspace{1cm} (3)

where $\theta$, $\alpha$, $\dot{\theta}$ and $\dot{\alpha}$ are the state variables of the plant.

### III. NATURAL FREQUENCY BASED SWING UP CONTROLLER

The swing-up controller swings the pendulum from the initial vertically downward position to vertically upward position.

Natural frequency based swing up controller: If pendulum has a length of $2l_p$ and its centre of mass is located at its geometric centre. Where $l_p$ is the length between rotating and mass centre of pendulum rod as shown in figure 2.

![Fig. 2: Rigid Pendulum](image)

Then natural frequency for small oscillations of the pendulum is given by

$$f = \frac{1}{2\pi} \sqrt{\frac{m g l_p}{I_p}}$$  \hspace{1cm} (4)

where $I_p = \frac{1}{2\pi} \sqrt{\frac{m g l_p}{I_p}}$ is the moment of inertia of pendulum about one end. Putting this value in equation 3.1 we will get

$$f = \frac{1}{2\pi} \sqrt{\frac{3g}{4 l_p}}$$  \hspace{1cm} (5)

$$T = \frac{1}{f}$$  \hspace{1cm} (6)

where $T$ is time period of oscillation which is fixed for individual pendulum. We can find the time period using above formula or experimentally. In experimentally we apply an impulse at pendulum and measure the time period. Based on this time period we can decide the direction in which we move the arm to feed the energy to the plant for the swing up. Time period of GRIP (2001) pendulum is 1.23 sec.

![Fig. 3: Implementation of natural frequency based swing-up controller in Simulink](image)

In this method no feedback from plant is required and decision is based on the time which is given by the clock in Simulink. Starting from the rest pendulum first changes its direction at $T/4$ second and after that every $T/2$ second direction change repeatedly. So using the clock we decide at which time reverse the direction of movement of arm to feed the energy in the plant for swing up the pendulum.

### IV. TS FUZZY BASED STABILIZATION CONTROLLER

The advantages of fuzzy logic controller (FLC) are their aptitudes to deal with nonlinearities and uncertainties so fuzzy controllers are helpful for nonlinear systems and multivariable systems. Fuzzy controllers consists of three stages. First is fuzzification stage which maps the input variable to the appropriate membership function. Second stage is the rule evaluation stage which invokes the appropriate rule and produces a result for each input, then combines the results of the rules. Last is the defuzzification stage which converts the combined result back into a specific control output value [4].

TS fuzzy use mathematical equations for the generation of the output. It expresses a non-linear system
into a linear system. The output control law ‘U’ can be expressed as:

\[ U = c_1 \theta + c_2 \dot{\theta} + c_3 \alpha + c_4 \dot{\alpha} + c_5 \]  

(7)

where \( c_1, c_2, c_3, c_4, c_5 \) are constants

\( \theta, \alpha, \dot{\theta}, \dot{\alpha} \) are the input variables. Two Gaussian type membership functions are chosen for each variable, one for negative input ‘N’ and another for positive input ‘P’. It leads to 16 rule formations, generating 16 different control signals for the control depending upon the value of inputs. Range for the position of both rotating arm and pendulum rod is been chosen -0.3 to 0.3, whereas range for the velocity of both the arms is taken -0.1 to 0.1. Value of constants for determination of the appropriate U is given in Table I. [1]

Table II describes the formation of the rule base. Rule base is designed considering the fact that a negative input will require negative force whereas a positive input will require a positive force to keep the pendulum rod in upright position. Magnitude of the force depends upon the position and velocity of the input. Zero input has not been considered for the simplicity of the rule base formation [4].

**TABLE I: Constants Value**

<table>
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<tr>
<th>U</th>
<th>( c_1 )</th>
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<th>( c_3 )</th>
<th>( c_4 )</th>
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**TABLE II: Rule base of TS fuzzy controller**

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<tr>
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<td>U11</td>
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<tr>
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<td>( \theta ), ( \dot{\theta} )</td>
<td>P:P</td>
<td>U4</td>
<td>U8</td>
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</tr>
</tbody>
</table>

**V. EXPERIMENTAL RESULTS**

Fig. 5 shows the Simulink model of complete experimental control system for RIP (GRIP2001). Block \(-\pi \sim \pi\) is used to change input signal between \(-\pi \sim \pi\) in 2 * \( \pi \) period. In real control there are two API module “Set Cart’s Acc and Vel” and “Get Pend’s Angle”. These modules are used to set the arm acceleration and get pendulum and arm position also respectively [2].

Googoltech motion controller card (GT400-SV) provides API Module “Set Cart’s Acc and Vel” and the same is taken as input to set the velocity and acceleration of rotating arm.

**Fig. 5: Experimental Simulink model of complete control system for (GRIP2001)**

**Fig. 6: Arm position and pendulum angle during swing-up using natural frequency based technique**
Experimental result in Fig. 6 shows that swing-up time is nearly 3.5 sec. We can reduce it further by increasing the gain of acceleration taken as input for plant. There is a switch for switching in between the swing-up and stabilizing controller according to the angle of pendulum from vertical position in real control block. In this experiment it is taken as 20 degree. During steady state arm position is around initial position and pendulum is at vertical upright position as shown in Fig. 7. During vertical upright pendulum angle is maintained at equilibrium point (α = 0 radian) with a 2% error band as shown in Fig. 8.

VI. CONCLUSION
Natural frequency based swing up is an open loop swing up technique and it can swing up the pendulum without any feedback from encoder mounted on it. An experimental result for swing up of pendulum using natural frequency based technique is nearly 3.5 second shown as in Figure 6. We can reduce it further by increasing the gain of acceleration taken as input for plant. Arm position is varying within 0.2 radian during swing-up. The main advantages of using TS fuzzy controller are it is computationally efficient and work with linear techniques. Liner equation coefficient can be easily calculated using state feedback technique. It is also well suited to mathematically analysis.

REFERENCES

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Books: