3- DOF Scara type Robot Manipulator using Mamdani Based Fuzzy Controller

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ABSTRACT
This paper aims 3-dof scara type robot manipulator using dynamic modeling. In this paper presents dynamic modeling of robot manipulator based on Lagrange Euler formulation using Mamdani based Fuzzy controller. The aim of designing fuzzy controller is to overcome the drawbacks of conventional PID controller because robot manipulators are nonlinear model so conventional PID controller cannot give adequate result. The performance of controller is calculated by Matlab/ Simulink in term of time response.

Keywords – 3-dof scara, Dynamic Modeling, Lagrange formulation, PID controller, Fuzzy controller

I. INTRODUCTION
In recent years, industrial and commercial systems with high efficiency and great performance have taken advantages of robot technology. Robot manipulator field is one of the interested fields in industrial, educational and medical applications. It works in unpredictable, hazard and inhospitable circumstances which human cannot reach. Therefore, modeling and analysis of the robot manipulators and applying control techniques are very important before using them in these circumstances to work with high accuracy. There are three main subsystems in robot manipulators: mechanical system, electrical system, and control system. One of the most vital and powerful issues in robotic fields is the control motion of the manipulator, because the robot operation must be accurate, without any effect in surrounding circumstances [1][2].

The scara model is a widely used industrial robot for congregation applications. Scara manipulator is a nonlinear and time variant system, the applied controllers are necessary to have variation capability where linear models are inadequate to complete high accuracy and various performance operations. To overcome the difficulty of traditional controllers, various control technique to improve the robot performance, like as neural network, sliding mode control, adaptive control and optimal control. Still, all these methods are wanting previous know- how of robot model [3].

In this paper presents the systematic procedure to establish the dynamic system of 3-dof scara type robot manipulator calculated with Lagrange Euler formulation. The scara robot manipulator has been simulated with Fuzzy controller.

II. DYNAMIC MODEL OF SCARA ROBOT MANIPULATOR
The dynamic equation of the manipulator describes the relationship between force, torque and motion. For multidimensional serial robot manipulator, the general form of n-link dynamic equation expressed in lagrangian form [3]:

\[
F = D(q(t))\ddot{q}(t) + C(q(t), \dot{q}(t))\dot{q}(t) + \tau_{friction} + G(q(t))
\]

Where: F denotes force or torque apply on joint n, t denotes time, q(t) denotes angle or distance of a joint, \(\dot{q}(t)\) and \(\ddot{q}(t)\) denotes joint velocity and joint acceleration, \(D(q) \in \mathbb{R}^n \times \mathbb{R}^n\) denotes inertial matrix, \(C(q, \dot{q}) \in \mathbb{R}^n \times \mathbb{R}^1\) denotes centrifugal and coriolis forces, \(G(q) \in \mathbb{R}^n \times \mathbb{R}^1\) denotes gravity loading, \(\tau_{friction}\) is joint friction.

Fig.1. Model of the Scara robot: joint connections and geometry of segments and joint positions

Using Lagrangian Euler formulation to obtain the dynamic solution depends on energy properties of manipulator:

\[
\frac{d}{dt} \left( \frac{\delta L}{\delta \dot{q}_k} \right) - \frac{\delta L}{\delta q_k} = \tau
\]

Where: \(\tau\) = generalized force (force F or torque T), \(q_k\) = generalized coordinates (displacement d or angle \(\Theta\))
L = \( K_e - P \)  Lagrange function,  \( K_e \) - kinetic energy,  \( P \) - potential energy

The torques applied on robot joints are:

\[
T_1 = \frac{\delta}{\delta t} \left( \frac{\delta L}{\delta \dot{\theta}_1} \right) - \frac{\delta L}{\delta \theta_1} = B_{11} \ddot{\theta}_1 + B_{12} \dot{\theta}_2 - B_{14} \dot{\theta}_1 \dot{\theta}_2 - B_{15} \dot{\theta}_1^2 \]  

(3)

\[
T_2 = \frac{\delta}{\delta t} \left( \frac{\delta L}{\delta \dot{\theta}_2} \right) - \frac{\delta L}{\delta \theta_2} = B_{21} \dot{\theta}_1 + B_{22} \dot{\theta}_2 - B_{24} \dot{\theta}_1^2 \]  

(4)

\[
T_3 = \frac{\delta}{\delta t} \left( \frac{\delta L}{\delta d_3} \right) - \frac{\delta L}{\delta d_3} = B_{33} d_3 - B_{34} \]  

(5)

Where:

\[
B_{11} = r_1^2 m_1 + J_1 + g_{r_1}^2 (J_m + J_{g_1}) + L_1^2 m_2 + (L_1^2 + r_2^2 + 2L_1 r_2 C_2) m_2 + J_2 + (L_1^2 + L_2^2 + 2L_1 L_2 C_2)(m_3 + m_m) \]

\[
B_{12} = (r_1^2 + L_1 r_2 C_2) m_2 + (L_1^2 + L_1 L_2 C_2)(m_3 + m_m) + J_2 \]

\[
B_{14} = 2L_1 S_2 [m_2 r_2 + (m_3 + m_m) L_2] \]

\[
B_{15} = L_1 S_2 [m_2 r_2 + (m_3 + m_m) L_2] \]

\[
B_{21} = (r_2^2 + L_1 r_2 C_2) m_2 + (L_1^2 + L_1 L_2 C_2)(m_3 + m_m) + J_2 \]

\[
B_{22} = r_2^2 m_2 + J_2 + g_{r_2}^2 (J_m + J_{g_2}) + L_2^2 (m_3 + m_m) \]

\[
B_{24} = L_1 S_2 [m_2 r_2 + (m_3 + m_m) L_2] \]

\[
B_{33} = \frac{4(J_m + J_{g_3})}{D^2} + (m_3 + m_m) \]

\[
B_{34} = (m_3 + m_m) g \]

The inertial matrix  \( D(q) \), Centrifugal and coriolis force  \( C(q, \dot{q}) \) and gravitational forces  \( G(q) \) is written as:

\[
D(q) = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{21} & B_{22} & 0 \\ 0 & 0 & B_{33} \end{bmatrix} \]

\[
C(q, \dot{q}) = \begin{bmatrix} -B_{14} \dot{\theta}_2 & -B_{15} \dot{\theta}_2 \\ B_{24} \dot{\theta}_1 & 0 \\ 0 & 0 \end{bmatrix} \]

\[
g(q) = \begin{bmatrix} 0 \\ 0 \\ -B_{34} \end{bmatrix} \]

In this model we can calculate joint angular acceleration as:

\[
\ddot{q} = D^{-1}(q)[F - C(q, \dot{q}) \ddot{q} - \tau_{friction} - G(q)] \]

When we integrate angular acceleration then we get angular velocity then integrate get angular displacement or position of each robot joint [3] [4].

III. ACTUATOR MODELING AND TRANSMISSION EQUATION

An actuator is a mechanical device that converts energy into motion. Various types of actuators are used such as hydraulic, pneumatic, electrical and mechanical. In robotics mostly DC Servo type motor are used. The equation of DC servo motor describe below:

\[
V_a = R i_a + e_a + L_a \frac{di}{dt} \]

(7)

Where back emf  \( e_a \) is proportional to angular speed of motor as:

\[
e_a = k_e \phi w_m = k_w w_m \]

(8)

The torque produced at the motor shaft is:

\[
T = k_e \phi i_a = k_T i_a = T_L + J_m \frac{dw_m}{dt} + b w_m \]

(9)

Transmission assigns simply to the gearbox that particular uses gears and gear trains to give speed and torque conversions from a rotating motor shaft. The transmitted torque can be computed as below:

\[
T = \frac{T_L}{g_r \eta} = > T_m \frac{w_m}{w_L} = T \frac{\theta_m}{\theta_L} \eta = T_L \]

(10)

For transmitted inertia,

\[
J = J_m + \frac{J_k}{\eta} \]

(11)

For the prismatic joint, the translational variable is computed as:

\[
w_m \frac{d}{dt} = d_3 \]

(12)
IV. CONTROLLER

Controlling robot manipulator is essential problem to guarantee the robot executing the desired task with minimum error. Control techniques used to enhance the efficiency of the robot manipulators in different fields starting from conventional controllers such as Proportional integral derivative (PID) controller to intelligent controller such as neural network (NN) and fuzzy controllers [4][5].

Here in this paper fuzzy controller is used for controlling the parameter of robot manipulator. The dynamic of the robot is modeled by nonlinear equations and FLC gives high flexibility for its many degree of freedom as number and type of membership functions, constructing rule base, fuzzification and defuzzification methods. Fuzzy systems are suitable for uncertainties and it deals with the controlled system as a black box. This means the mathematical model of the system is difficult or too complex to derive.

The fuzzy controller consist of four main components: fuzzification, rule base, inference mechanism and defuzzification methods.

In fuzzification is the first block inside the controller, which scale the input crisp value e and ∆e into normalized universe of discourse U then converts each crisp input to degrees of membership function $\mu_A(u)$[6].

Various type of membership functions are used in fuzzy like as Gaussian, triangular, trapezoidal etc. here in this paper triangular membership function is used. Each membership function is defined by linguistic variable, in this paper input membership function has five linguistic input variables such as negative big (NB), negative medium (NM), negative small (NS), zero (Z), positive small (PS), positive medium (PM), positive big (PG) and output membership function has three linguistic variable such as small (S), medium (M), big (B). The membership function shown in figure

![Membership function](image1)

Rule base is the core of the FLC; it is a set of rules in the form of IF-THEN statement that describe the state and the behavior of the control system. The rule base used to control the system as shown in table.

<table>
<thead>
<tr>
<th>$\dot{e}/e$</th>
<th>NB</th>
<th>NM</th>
<th>NS</th>
<th>Z</th>
<th>PS</th>
<th>PM</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>NB</td>
<td>B</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td>B</td>
</tr>
<tr>
<td>NM</td>
<td>B</td>
<td>B</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td>B</td>
<td>B</td>
</tr>
<tr>
<td>NS</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>S</td>
<td>B</td>
<td>B</td>
<td>B</td>
</tr>
<tr>
<td>Z</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>B</td>
</tr>
<tr>
<td>PS</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>S</td>
<td>B</td>
<td>B</td>
<td>B</td>
</tr>
<tr>
<td>PM</td>
<td>B</td>
<td>B</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td>B</td>
<td>B</td>
</tr>
<tr>
<td>PB</td>
<td>B</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td>B</td>
</tr>
</tbody>
</table>

Inference mechanism is obtaining the relevant control rule at the current time and then decides what the controller output should be. In fuzzy controller most widely two type of inference mechanism are used Mamdani method “Min-Max” and Larson method “Product –Max” here in this paper Min-Max type are used.

Defuzzification method is the final stage of the fuzzy logic control. Defuzzification method converts the resulting fuzzy set into crisp values that can be sent to the plant as a control signal. Various methods are used for defuzzification; here in this paper centroid method are used.

V. SIMULATIONS AND RESULTS

The specification of Scara robot is define as; masses of main arm $m_1=1.7$ Kg, fore arm $m_2=0.83$ Kg and $m_3=0.35$ Kg, the length of main arm $l_1=0.35$m, fore arm $l_2=0.25$m and quill $d_3=0.085$m, the moment of inertia of main and fore arm are $J_1=0.088$ Kg m$^2$ and $J_2=0.0125$ Kg m$^2$. The specification of Dc servo motor defines below:

- $J_m=0.000062$ Kg m$^2$, $K_i= 0.0245$ V/rad/s, $K_t= 0.0245$ Nm/A, $R= 5$ ohm, $L_a= 0.27$ mH, $g_{r1} = 90$, $g_{r2} = 220$, $D= 0.03$ m

![Step response of Fuzzy controller](image2)
In fig.3 and fig 4 shows the output of positions arms using Fuzzy controller. Fuzzy controller has step input 1, 2 and 4. The table of performance specification defines below

**TABLE II. Response parameters of system based on fuzzy controller**

<table>
<thead>
<tr>
<th>Step Response</th>
<th>Rise time</th>
<th>Peak overshoot</th>
<th>Settling time</th>
<th>Steady state error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arm 1</td>
<td>1.4</td>
<td>0</td>
<td>1.7</td>
<td>0</td>
</tr>
<tr>
<td>Arm 2</td>
<td>1.6</td>
<td>0</td>
<td>1.9</td>
<td>0</td>
</tr>
<tr>
<td>Arm 3</td>
<td>0.7</td>
<td>0</td>
<td>0.9</td>
<td>0</td>
</tr>
<tr>
<td>Exponential Response</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Arm 1</td>
<td>1.4</td>
<td>0</td>
<td>1.6</td>
<td>0</td>
</tr>
<tr>
<td>Arm 2</td>
<td>1.5</td>
<td>0</td>
<td>1.7</td>
<td>0</td>
</tr>
<tr>
<td>Arm 3</td>
<td>1.0</td>
<td>0</td>
<td>1.2</td>
<td>0</td>
</tr>
</tbody>
</table>

**VI. CONCLUSION**

In this paper, dynamic modeling of 3-dof scara type robot manipulator using Lagrange Euler formulation and controlling the parameter of manipulator using fuzzy controller. Robot manipulator is non-linear model and time variant. Conventional controllers are not able to give desired response. Fuzzy controller can be applied linear or non-linear model, the output response of fuzzy controller as shown in figures has no peak overshoot and steady state error. Fuzzy controller can be tuned offline and online. Based on simulation response, fuzzy controller gives satisfactory results.

**REFERENCES**

Books:

Papers:

**SPECIFICATION**

- \( L_1 \), \( L_2 \): Lengths of main arm and fore arm (m)
- \( J_m \): Motor inertia (Kg m²)
m₁, m₂ - Masses of the main arm and fore arm (Kg)
J₁, J₂ - Moment of inertias of the main and fore arm (Kg m²)
Vₐ - Armature voltage (volts)
R - Armature resistance (Ω)
T - Torque of motor
B - Damping ratio
Kₜ - Torque constant
Θ₁, Θ₂, d₃ - Angular displacement and translational Displacement
Θ̇₁, Θ̇₂, Θ̇₃ - Angular velocity and translational velocity
Θ̈₁, Θ̈₂, Θ̈₃ - Angular acceleration and translational Acceleration
D - Warm wheel diameter
g₉ - Gear ratio
J₁, J₂ - Inertias of gear box
η - Efficiency of the system
Θₘ - Angular position of motor (rad)
Θₗ - Load angular position