A Markov Decision Model to Optimize Hotel Room Occupancy under Stochastic Demand

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ABSTRACT
Hotels continually face the challenge of planning and managing room occupancy within an environment of demand uncertainty. In this paper, an optimization method for allocating hotel rooms to occupants is proposed. The objective of the model is to optimally admit customers in order to maximize revenue. Adopting a Markov decision process approach, the states of a Markov chain represent possible states of demand for hotel room occupancy. The decision of whether or not to admit additional occupants is made using dynamic programming over a finite period planning horizon. The approach demonstrates the existence of an optimal state-dependent admission policy for occupants as well as the corresponding hotel room revenue.

Keywords - Hotel room occupancy, Markov, Optimize, Stochastic demand

1. INTRODUCTION
Optimal planning and allocation of hotel rooms to occupants is still a great challenge in hotel room administration and revenue management. This is a considerable challenge when the demand for rooms follows a stochastic trend. It requires proper understanding of the environment with which the hotel is operating and the development of a vision regarding future room allocation decisions. Proper planning and management must therefore ensure that the requirements of room occupants are matched with room availability so that hotel customer care is not hindered by lack of rooms or the hotel does not have idle, excess rooms on hand. In order to achieve this goal, two major problems are usually encountered:

(i) Determining the most desirable period during which to admit additional occupants
(ii) Determining the optimal total revenue for managing hotel room administration that corresponds to the best occupant admission policy given a periodic review room allocation strategy when demand for rooms is uncertain

It is essential for the hotel to gauge and know its reasonable or optimum capacity as no single level can sustain room occupancy and revenue collection.

In this paper, a mathematical model is developed whose goal is to optimize the admission decisions of occupants and the total revenue associated with the admission decisions taken under demand uncertainty of rooms.

The paper is organized as follows. After reviewing the previous work done, the mathematical model is described where consideration is given to the process of estimating the model parameters. The model is then solved and applied to a special case study. Thereafter, final remarks follow.

2. LITERATURE REVIEW
El Gayar [1] presented a model of an integrated framework for hotel room revenue maximization. The model treats the shortcomings in existing systems by extending the optimization techniques that exist for hotel revenue management to address group reservations. The model also includes integrality constraints and forecasted demand arrivals generated from data. According to Baker and Riley[2] it is noted that most practical approaches to hotel productivity is from the vantage point of management ability to forecast demand and against this, assess performance. In most cases, it leads to supply-demand mismatch. In the paper presented by Rajopadhye[3],the problem of forecasting uncertain demand for hotel rooms can be analyzed by Holt-Winters method. This is part of the revenue management problem whose objective is to maximize the revenue by making decisions regarding when to make rooms available for customers and at what price.Badinelli [4] examines an optimal dynamic policy for hotel yield management .This is done allowing for general demand patterns and a policy that is based on time and the number of vacancies. The policy incorporates both revealed price and hidden price market behavior and it is shown that this formulation has simple closed-form solutions that can
efficiently be computed. In a similar context, Qu[5] identified the important factors that influence the hotel room supply and demand and their overall impact on the Hong-Kong hotel industry. Time series data and a simultaneous equations econometric model is employed. Empirical results indicate that hotel room price and tourist arrivals are significant factors driving the demand for hotel rooms. Lai [6] also addressed hotel revenue management under uncertain environment using stochastic programming to capture the randomness of the unknown demand. Strategies are also discussed for hotel revenue management to take into account risk-trade-off, different pricing policies, cancellations and no show, early check outs, extended stay and overbooking. In the article presented by Liu and Lu[7], a Hot-winters process method is used to forecast room demands in future periods. A hybrid optimization model is then built for hotel yield management and a branch and bound method mixed greedy algorithm is developed to solve the problem. Liu[8] further presented a stochastic optimization model for hotel revenue management with multiple day stays under uncertain environment. Since a decision maker may face several scenarios when renting out rooms, a semi-absolute deviation model is used to measure the risk of hotel revenue, and only the risk of falling below the expected revenue. Rothstein [9] studied the hotel overbooking problem as a Markovian sequential decision process. The paper deals with the problems of hotel overbooking and determination of scientific overbooking policies. The model is presented, based on actual sample hotel data. Because of the significance of cancellation phenomena in the sample, the model prescribes a substantial amount of overbooking. In the article presented by Zaakhary[10], a Monte Carlo simulation approach for hotel arrival and occupancy is forecasted. In this approach, simulation is done for hotel reservations forward in time and the future Monte Carlo paths yield forecast densities. Parameters are estimated from historical data and the reservation process is simulated forward with its constituent processes, such as reservation arrivals, cancellations, length of stay, no show ups, group reservations, seasonality, trend etc.

3. MODEL FORMULATION

3.1 Notation and Assumptions

- \( i,j \) = States of demand
- \( A \) = Very high demand
- \( B \) = High demand
- \( C \) = Moderate demand
- \( D \) = Low demand
- \( E \) = Very low demand

- \( n,N \) = Stages
- \( S \) = Admission policy
- \( N^s \) = Customer matrix
- \( N^s_{ij} \) = Number of customers
- \( Q^s \) = Demand transition matrix
- \( R^s \) = Revenue matrix
- \( W^s_i \) = Expected revenue
- \( m^s_i \) = Accumulated revenue

- \( i,j \in \{A,B,C,D,E\} \)
- \( S \in \{0,1\} \)
- \( n=1,2, \ldots, N \)

We consider a hotel whose demand for rooms can be classified depending upon the percentage of room occupancy over a fixed planning horizon. Demand for rooms can be classified as very high (denoted by state A), high (denoted by state B), moderate (denoted by state C), low (denoted by state D), and very low (denoted by state E). The representation assumes the following correspondence between room occupancy and the states of the chain:

<table>
<thead>
<tr>
<th>Room Occupancy (%)</th>
<th>State of Demand</th>
<th>State Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>85-100</td>
<td>Very High</td>
<td>A</td>
</tr>
<tr>
<td>70-84</td>
<td>High</td>
<td>B</td>
</tr>
<tr>
<td>55-69</td>
<td>Moderate</td>
<td>C</td>
</tr>
<tr>
<td>40-54</td>
<td>Low</td>
<td>D</td>
</tr>
<tr>
<td>0-39</td>
<td>Very Low</td>
<td>E</td>
</tr>
</tbody>
</table>

The demand of any such period is assumed to depend on the demand of the preceding period. The transition probabilities over the planning horizon from one demand state to another may be described by means of a Markov chain. Suppose one is interested in determining an optimal course of action, namely to admit additional customers (a decision denoted by \( S=1 \)) or not to admit additional customers (a decision denoted by \( S=0 \)) during each time period over the planning horizon, where \( S \) is a binary decision variable. Optimality is defined such that the maximum revenue is accumulated at the end of \( N \) consecutive time periods spanning the planning horizon under consideration. In this paper, a two-period \((N=2)\) planning horizon is considered.

3.2 Finite -period dynamic programming problem formulation

Recalling that the demand can be in one of the states A, B, C, D and E the problem of finding an optimal
admission policy may be expressed as a finite period dynamic programming model.

Let \( f_n(i) \) denote the optimal expected revenue accumulated during periods \( n,n+1,\ldots,N \) given that the state of the system at the beginning of period \( n \) is \( i \in \{A,B,C,D,E\} \). The recursive equation relating \( f_n \) and \( f_{n+1} \) is

\[
f_n(i) = \max_{j \in \{A,B,C,D,E\}} (Q^S_{ij} R^S_{ij} + f_{n+1}(j)),
\]

\[
f_n(i) = \max_{j \in \{A,B,C,D,E\}} (Q^S_{ij} R^S_{ij} + f_{n+1}(j)),
\]

This recursive equation may be justified by noting that the cumulative revenue \( R^S_{ij} \) resulting from reaching state \( j \in \{A,B,C,D,E\} \) at the start of period \( n+1 \) from state \( i \in \{A,B,C,D,E\} \) at the start of period \( n \) occurs with probability \( Q^S_{ij} \).

Clearly,

\[
W^S_i = \sum_{j \in \{A,B,C,D,E\}} Q^S_{ij} R^S_{ij},
\]

\[
W^S_i = \sum_{j \in \{A,B,C,D,E\}} Q^S_{ij} R^S_{ij},
\]

and hence the dynamic programming recursive equations

\[
f_n(i) = \max_{S \in \{0,1\}} (W^S_i + Q^S_{ij} f_{n+1}(i)),
\]

\[
f_N(i) = \max_{S \in \{0,1\}} W^S_i
\]

\[
N=1,2,\ldots,N
\]

\[
i,j \in \{A,B,C,D,E\}
\]

### 3.3 Computing \( Q^S \)

The demand transition probability from state \( i \in \{A,B,C,D,E\} \) to state \( j \in \{A,B,C,D,E\} \) given admission policy \( S \in \{0,1\} \) may be taken as the number of customers observed with demand initially in state \( i \) and later with demand changing to state \( j \) divided by the number of customers over all states.

That is

\[
Q^S_{ij} = N^S_{ij}/[N^S_{iA} + N^S_{iB} + N^S_{iC} + N^S_{iD} + N^S_{iE}]
\]

\[
Q^S_{ij} = N^S_{ij}/[N^S_{iA} + N^S_{iB} + N^S_{iC} + N^S_{iD} + N^S_{iE}]
\]

\[
i,j \in \{A,B,C,D,E\}, \quad S \in \{0,1\}
\]

### 4. COMPUTING AN OPTIMAL ADMISSION POLICY

The optimal admission policy and revenue generated are found in this section for each period separately.

#### 4.1 Optimization during period 1

The optimal admission policy and revenue generated during period 1 is

\[
S = \begin{cases} 1 & \text{if } W^1_i > W^0_i \\ 0 & \text{if } W^1_i \leq W^0_i \end{cases}
\]

\[
f_n(i) = \begin{cases} W^1_i & \text{if } S = 1 \\ W^0_i & \text{if } S = 0 \end{cases}
\]

\[
i,j \in \{A,B,C,D,E\}
\]

#### 4.2 Optimization during period 2

Using (2) and (3), and recalling that \( m^S_i \) denotes the already accumulated revenue at the end of period 1 as a result of decisions made during that period, it follows that

\[
m^S_i = W^1_i + \sum_{j \in \{A,B,C,D,E\}} Q^S_{ij} \max(W^1_j, W^0_j) + Q^S_{ij} \max(W^1_j, W^0_j) + Q^S_{ij} \max(W^1_j, W^0_j) + Q^S_{ij} \max(W^1_j, W^0_j)
\]

\[
W^S_i = \sum_{j \in \{A,B,C,D,E\}} Q^S_{ij} f_1(i) + Q^S_{ij} f_2(i)
\]

\[
W^S_i = \sum_{j \in \{A,B,C,D,E\}} Q^S_{ij} f_1(i) + Q^S_{ij} f_2(i)
\]

Therefore the optimal admission policy and revenue generated during period 2 is

\[
S = \begin{cases} 1 & \text{if } m^1_i > m^0_i \\ 0 & \text{if } m^1_i \leq m^0_i \end{cases}
\]

\[
f_2(i) = \begin{cases} m^1_i & \text{if } S = 1 \\ m^0_i & \text{if } S = 0 \end{cases}
\]

\[
i,j \in \{A,B,C,D,E\}
\]

### 5. CASE STUDY

In order to demonstrate use of the model in § 3-4, a real case application from Holiday Express Hotel in Uganda is presented in this section. The demand for hotel rooms fluctuate every week; and the hotel wants to avoid admitting excess occupants when all rooms are occupied to full capacity or reject admissions when unoccupied rooms are available under different states of demand: Very high (state A), High (state B), Moderate (state C), Low (state D), and Very low (state E). Decision support is therefore sought in terms of an optimal admission policy for occupants and the associated revenue in a two-week planning period.

#### 5.1 Data Collection

Samples of customers for room rentals and the associated revenue collected (in hundred dollars) over twelve weeks were noted under different state transitions and admission policies at Holiday Express Hotel.
The following data was captured when additional occupants were admitted (S=1):

<table>
<thead>
<tr>
<th>State Transitation (i,j)</th>
<th>Admission Policy (S)</th>
<th>Customers N_{i,j}^S</th>
<th>Revenue R_{i,j}^S</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA</td>
<td>1</td>
<td>20</td>
<td>50</td>
</tr>
<tr>
<td>AB</td>
<td>1</td>
<td>10</td>
<td>25</td>
</tr>
<tr>
<td>AC</td>
<td>1</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>AD</td>
<td>1</td>
<td>3</td>
<td>17</td>
</tr>
<tr>
<td>AE</td>
<td>1</td>
<td>2</td>
<td>15</td>
</tr>
<tr>
<td>BA</td>
<td>1</td>
<td>10</td>
<td>18</td>
</tr>
<tr>
<td>BB</td>
<td>1</td>
<td>15</td>
<td>40</td>
</tr>
<tr>
<td>BC</td>
<td>1</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>BD</td>
<td>1</td>
<td>4</td>
<td>18</td>
</tr>
<tr>
<td>BE</td>
<td>1</td>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>CA</td>
<td>1</td>
<td>8</td>
<td>15</td>
</tr>
<tr>
<td>CB</td>
<td>1</td>
<td>6</td>
<td>19</td>
</tr>
<tr>
<td>CC</td>
<td>1</td>
<td>20</td>
<td>18</td>
</tr>
<tr>
<td>CD</td>
<td>1</td>
<td>4</td>
<td>14</td>
</tr>
<tr>
<td>CE</td>
<td>1</td>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>DA</td>
<td>1</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>DB</td>
<td>1</td>
<td>8</td>
<td>14</td>
</tr>
<tr>
<td>DC</td>
<td>1</td>
<td>4</td>
<td>18</td>
</tr>
<tr>
<td>DD</td>
<td>1</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>DE</td>
<td>1</td>
<td>7</td>
<td>11</td>
</tr>
<tr>
<td>EA</td>
<td>1</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>EB</td>
<td>1</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>EC</td>
<td>1</td>
<td>5</td>
<td>12</td>
</tr>
<tr>
<td>ED</td>
<td>1</td>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>EE</td>
<td>1</td>
<td>18</td>
<td>20</td>
</tr>
</tbody>
</table>

Similarly, when additional occupants were not admitted (S=0), the following data was captured:

<table>
<thead>
<tr>
<th>State Transitation (i,j)</th>
<th>Admission Policy (S)</th>
<th>Customers N_{i,j}^S</th>
<th>Revenue R_{i,j}^S</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA</td>
<td>0</td>
<td>22</td>
<td>38</td>
</tr>
<tr>
<td>AB</td>
<td>0</td>
<td>3</td>
<td>20</td>
</tr>
<tr>
<td>AC</td>
<td>0</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>AD</td>
<td>0</td>
<td>5</td>
<td>16</td>
</tr>
<tr>
<td>AE</td>
<td>0</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>BA</td>
<td>0</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>BB</td>
<td>0</td>
<td>16</td>
<td>40</td>
</tr>
<tr>
<td>BC</td>
<td>0</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>BD</td>
<td>0</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>BE</td>
<td>0</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>CA</td>
<td>0</td>
<td>8</td>
<td>6</td>
</tr>
</tbody>
</table>

5.2 Solution Procedure for the room occupancy problem

When additional occupants were admitted (S=1),

\[ Q^1 = \begin{bmatrix} 0.500 & 0.250 & 0.125 & 0.075 & 0.050 \\ 0.250 & 0.375 & 0.200 & 0.100 & 0.075 \\ 0.150 & 0.200 & 0.100 & 0.375 & 0.175 \\ 0.225 & 0.150 & 0.125 & 0.050 & 0.450 \end{bmatrix} \]

\[ R^1 = \begin{bmatrix} 50 & 25 & 10 & 17 & 13 \\ 18 & 40 & 10 & 18 & 15 \\ 12 & 14 & 18 & 20 & 11 \\ 8 & 10 & 12 & 18 & 20 \end{bmatrix} \]

When additional occupants were not admitted (S=0),

\[ Q^0 = \begin{bmatrix} 0.550 & 0.200 & 0.075 & 0.125 & 0.050 \\ 0.225 & 0.400 & 0.200 & 0.125 & 0.050 \\ 0.200 & 0.125 & 0.450 & 0.150 & 0.025 \\ 0.175 & 0.250 & 0.075 & 0.400 & 0.100 \\ 0.250 & 0.125 & 0.075 & 0.100 & 0.450 \end{bmatrix} \]

\[ R^0 = \begin{bmatrix} 38 & 20 & 12 & 16 & 20 \\ 10 & 40 & 8 & 10 & 18 \\ 8 & 12 & 20 & 18 & 12 \\ 8 & 10 & 12 & 24 & 20 \end{bmatrix} \]

Using (2), the expected weekly revenue generated (in hundred dollars) is determined for each admission policy and results are summarized in Table 1 below:
Using (2) and (3) the accumulated revenue (in hundred dollars) is determined for each admission policy and results are summarized in Table 2 below:

### Table 2

<table>
<thead>
<tr>
<th>State (i)</th>
<th>Accumulated Revenue $W_s^i$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S=1$</td>
</tr>
<tr>
<td>A</td>
<td>43.64</td>
</tr>
<tr>
<td>B</td>
<td>48.07</td>
</tr>
<tr>
<td>C</td>
<td>35.12</td>
</tr>
<tr>
<td>D</td>
<td>36.09</td>
</tr>
<tr>
<td>E</td>
<td>33.72</td>
</tr>
</tbody>
</table>

5.3 The optimal admission policy and revenue

**Week 1:**
Since 34.4 > 28.3, it follows that $S=1$ is an optimal admission policy for week 1 with associated expected revenue of $3400 for the case of Very High demand. Since 24.4 > 21.9, it follows that $S=1$ is an optimal admission policy for week 1 with associated expected revenue of $2440 for the case of High demand. Since 22.9 > 11.9 it follows that $S=1$ is an optimal admission policy for week 1 with associated expected revenue of $2290 for the case of moderate demand. Since 14.3 > 9.8, it follows that $S=0$ is an optimal admission policy for week 1 with associated expected revenue of $1430 for the case of Low demand. Since 15.6 > 9.9, it follows that $S=0$ is an optimal admission policy for week 1 with associated expected revenue of $1560 for the case of very low demand.

**Week 2:**
Since 45.67 > 43.64, it follows that $S=0$ is an optimal admission policy for week 2 with associated accumulated revenue of $4567 for the case of Very High demand. Since 49.05 > 48.07, it follows that $S=0$ is an optimal admission policy for week 2 with associated accumulated revenue of $4905 for the case of High demand. Since 35.38 > 35.12 it follows that $S=0$ is an optimal admission policy for week 2 with associated accumulated revenue of $3538 for the case of moderate demand. Since 36.09 > 34.69, it follows that $S=1$ is an optimal admission policy for week 2 with associated accumulated revenue of $3609 for the case of Low demand. Since 33.72 > 32.86, it follows that $S=1$ is an optimal admission policy for week 2 with associated accumulated revenue of $3372 for the case of very low demand.

### 6. CONCLUSION

A hotel room occupancy model is presented in this paper. The model determines an optimal admission policy for hotel occupants and the associated revenue under stochastic demand. The decision of whether or not to admit additional occupants is modeled as a multi-period decision problem using dynamic programming over a finite period planning horizon. As a revenue maximization strategy in hotel room administration, computational efforts of using Markov decision process provide promising results. It would however be worthwhile to extend the research and examine the behavior of admission policies under non stationary demand conditions. In the same spirit, the model raises a number of salient issues to consider: Price disruptions of rooms in the hotel industry as well as overbooking and room reservations for customers. Finally, special interest is sought in further extending the model by considering admission policies within the context of Continuous Time Markov Chains (CTMC).

### REFERENCES


