Effect of Ramp Input on RLC Interconnect In Modern VLSI Design

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ABSTRACT

In the contemporary years there has been a significant research in finding out the closed form expression for the delay of an RLC circuit. However, there are ample of techniques and formulae that hypothesize a step excitation, and later on find a suitable extension to the inputs such as the ramp inputs (assume a saturated ramp). There are some of the experiments that do consider ramp inputs but do not present a closed form formula that works for a wide range of input slews. By taking into the account the impulse responses of linear circuit as a Probability Distribution Function (PDF), Elmore was the first through which the value was estimated of the interconnect delay. A number of approaches were made into and have been proposed after the well known metric such as the Elmore delay metric Elmore like h-gamma, AWE , PRIMO , WED , D2M and many more. and are successful and are proven to be more precise than the Elmore delay metric. But the only bottleneck served with this is that they have computational complexity when used in total IC design process. This paper primarily presents a closed form formula for on-chip VLSI RLC interconnects Slew. Slew Metric can be computed and obtained by matching the circuit moments of the Beta Distribution Function. To determine the interconnect slew of the RLC linear circuit we need to match the three circuit moments. This method is much more efficient involving less computational complexity. The model proves to be very much accurate and precise and is completely justified when compared with the SPICE simulations. This paper basically enumerates an approach for adjusting and fitting the moments of the impulse response to probability density functions so as to determine the delay accurately at a very early stage. For RLC trees it is demonstrated that the inverse gamma function provides a provably stable approximation. PERI is one of the techniques (Probability distribution function Extension for Ramp Inputs) that stretches and extends the delay metrics for the input such as the ramp inputs to the more realistic non-step inputs.

Keywords: Moment Matching, On-Chip Interconnect, Probability Distribution function, Cumulative Distribution function, Delay calculation, Slew Calculation, Beta Distribution, VLSI.

I. INTRODUCTION

Since the integrated VLSI circuits have increasing complexity and the modelling and simulation of various VLSI macro models such as the interconnects have high accuracy and efficiency. The sub-quarter micron IC technologies have proved to give vast dramatic changes in the methodologies of manufacturing in integrated circuits and systems. The most important factor is the feature sizing of the integrated circuits that is scaled well below 0.18 microns, meanwhile the active device counts are reaching hundreds of millions [1]. The amount of interconnect among the devices tends to grow super linearly along with the transistor counts, and the chip area is often limited by the physical interconnect area. Due to these interconnect area limitations, the interconnect dimensions are scaled with the devices whenever possible. In addition, to provide more wiring resources, IC’s now accommodate numerous metallization layers, with more to come in the future. These advances in technology that result in scaled, multi-level interconnects may address the wireability problem, but in the process creates problems with signal integrity and interconnect delay. This paper basically proposes and focuses on an extension of Elmore’s approximation [2] so as to include matching of higher order moments of the probability density function. Specifically, using a time-shifted incomplete Gamma function approximation [3] for the impulse responses of RC trees, the three parameters of this model are fitted by matching the first three central moments such as the variance , mean and the skewness, which is almost equivalent to matching the first two moments of the circuit response (m1,m2). Most importantly, it is noticed that the gamma function is proven to be realizable and stable for the moments if the RLC tree. Once the moments are fitted to specify and characterize the Gamma function [4]. Once the moments are fitted to characterize the Gamma function, slew and the step response delay are obtained in closed form expression.

In this experiment, we have used one of the techniques known as the PERI technique[13] for extending or stretching the delay metric for a step input into a delay
metric for a ramp input for RLC trees that is applicable over all input slews. An important feature of this method is that the delay metric reduces to the Elmore delay of the circuit under the limiting case of an infinitely slow ramp.

The remaining part of the paper is organized as follows. Section 2 presents pertinent background information relating probability and circuit theory. Section 3 presents the techniques for extending step delay and slew metrics to ramp inputs. Section 4 will focus on the experimental results that both validate and enhance the effectiveness of our approach. Finally, we will be concluding the paper in Section 5.

II. BASIC THEORY

II.1 Moments of a Linear Circuit Response

Let \( h(t) \) be a circuit impulse response in the time domain and let \( H(s) \) be the corresponding transfer function. By definition, \( H(s) \) is the Laplace transform of \( h(t) \) [6],

\[
H(s) = \int_{0}^{\infty} h(t)e^{-st} dt \quad \ldots \ldots (1)
\]

Applying a Taylor series expansion of \( e^{-st} \) about \( s = 0 \) yields,

\[
H(s) = \sum_{i=0}^{\infty} \frac{(-1)^i s^i}{i!} \int_{0}^{\infty} s^i h(t) dt
\]

Now apply different limits for finding \( m_0, m_1, m_2, m_3 \ldots \ldots m_n \)

\[
m_0 = \int_{0}^{1} th(t) dt
\]

\[
m_0 = h(t) \int_{0}^{1} t dt
\]

After solving \( m_0 \) we get

\[
m_0 = \frac{h(t)}{2}
\]

Similarly,

\[
m_1 = 2h(t)
\]

\[
m_2 = \frac{9h(t)}{2}
\]

\[
m_3 = 8h(t) \quad \text{And so on…}
\]

Now transfer function \( h(t) \) can be expressed as,

\[
H(s) = m_0 + m_1s + m_2s^2 + m_3s^3 + \ldots \ldots
\]

\[
H(s) = \frac{h(t)}{2} + 2h(t) + \frac{9h(t)}{2} + 8h(t) + \ldots \ldots \ldots (3)
\]

II.2 Central Moment

Central moments are generally distribution theory concepts. With the help of Elmore’s distribution function analogy, we can use them to describe the properties of Elmore delay approximation.

Consider the given moment definition

\[
p_q = \frac{(-1)^q}{q!} \int_{0}^{\infty} q h(t) dt
\]

This means impulse is given response by [9,10]

\[
\gamma = \frac{\int_{0}^{\infty} th(t) dt}{\int_{0}^{\infty} h(t) dt} = -\frac{m_1}{m_0}
\]

Few central moments can be expressed in terms of circuit moment [11]

\[
\gamma_0 = m_0,
\]

\[
\gamma_1 = 0,
\]

\[
\gamma_2 = 2m_2 - \frac{m_1^2}{m_0}
\]

\[
\gamma_3 = -6m_3 + 6\frac{m_1m_2}{m_0} - 2\frac{m_3}{m_0} \ldots \ldots \ldots \ldots (4)
\]

\((\gamma_0)\) is the area under the curve, which is unity.

\((\gamma_2)\) is the variance of the distribution function which measure spread of the curve from the centre.

\((\gamma_3)\) is the measure of skewness of the distribution function.

II.3 Higher Central Moments in RLC Trees

The second and third central moments are always positive for RC tree impulse responses [10]. The positiveness of the second order central moment is obvious from its definition.
\[ \alpha_z = \int_0^\infty (t - \alpha) h(t) dt \]

The impulse response, \( h(t) \), at any node in an RLC tree is always positive. Hence the second central moment \( \mu_2 \) is always positive.

### III. PROPOSED MODEL

The interconnect delay was approximated and based on the analogy of non-negative impulse responses and the PDF i.e. probability distribution function. The Beta distribution provides a good representation of RC tree impulse responses since it has good “coverage” of bell shaped curves.

Beta distribution is a two-parameter continuous function given by the following formula [11],

\[ B(r, s) = \int_0^1 v^{r-1}(1-v)^{s-1} dv \]

for One can easily generate the PDF of Beta distribution by using Mat lab. Figure 1 given below is the Beta distribution PDF with \( P(a, b) \) on y-axis and X on x-axis which is generated in Mat lab 7.5.0 different values of constants \( r \) and \( s \).

![Beta distribution PDF](image)

**Fig.1.** Beta distribution PDF for Different values of constants \( r \) and \( s \)

Now the expressions of Mean (\( \mu \)) and Variance (\( \sigma^2 \)) and Skewness

\[ \alpha_3 \] [13] is given by

\[ \mu = \frac{R}{r + s} \]

\[ \sigma^2 = \frac{Rs}{(r + s)^2 (r + s + 1)} \]

\[ S.D = \sigma_1 = \frac{rs}{\sqrt{(r^2 + s^2 + 2rs)(r + s + 1)}} \]

\[ \gamma_3 = \frac{2(s-r)(r+s+1)^{1/2}}{(r+s+2)(rs)^{1/2}} \]

The probability density function for the beta distribution is defined [11] as below respectively.

\[ \alpha = -m_1 = \frac{r}{r + s} \]

\[ \alpha_3 = -6m_3 + 6m_1 m_2 - 2m_1^3 = \frac{2(s-r)(r+s+1)^{1/2}}{(r+s+2)(rs)^{1/2}} \]

For \( r>1, s>1 \)

Now deriving the expressions for constants \( a \) and \( b \) from (15) and (16) in terms of \( m_1 \) and \( m_2 \), we get

\[ r = \frac{m \{n - (n^2 - 4mp)^{1/2}\}}{2m(1 + m_1)} \]

\[ s = \frac{-n + (n^2 - 4mp)^{1/2}}{2m} \]

Where \( m, n \) and \( p \) have been calculated and given by the formulae,

\[ m = 2m_1^4 - 6m_2^2 m_2 + 6m_1 m_2 \]

\[ n = 4m_1^3 + 4m_1^4 - 12m_1^2 m_2 - 12m_2^2 m_2 + 12m_1^2 m_3 + 12m_1 m_3 - 4m_1 - 2 \]

\[ p = -2m_1^2 - 3m_1 - 1 \]

\[ I(p,q) = \frac{1}{Q(p,q)} \int_0^p x^{p-1}(1-x^q)dx \]

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\[ \text{as the Cumulative n as shown below} \]

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This CDF satisfies the following conditions:

\[ 0 \leq I_t \leq 1 \]

\[ \lim_{t \to 0} (I_t = 0) \quad \lim_{t \to 1} (I_t = 1) \]

Since we get the values of interconnect delay in terms of nanoseconds we can approximate

\[ (1 - \theta)^q = 1, \quad (25) \]

\[ 1 = \sum_{n=0}^{\infty} \frac{Q(p + 1, n + 1)}{B(p + q, n + 1)}, \quad \sum_{n=0}^{\infty} \approx 1 \]

The required expression for Beta Slew Metric (BSM) can be obtained by using the following formula

\[ BSM = T_{HI} - T_{LO} \] (32)

From (30) and (31) we get the expression of BSM as

\[ BSM = [0.9pQ(p, q)]^p - [0.1pQ(p, q)]^p \] (33)

Simplifying function [11] and substituting in B (a, b) to get

\[ I_t(p, q) = \frac{pQ(p, q)}{B(p + q, p + q)} \] (27)

By using these two approximations (25) and (26) we can simplify the required CDF as

\[ I_t(p, q) = \frac{pQ(p, q)}{B(p + q, p + q)} \] (27)

Now, let \( T_{LO} \) and \( T_{HI} \) be 10% and 90% Delay points respectively. Matching these points to CDF (27) we get Substituting the values of a and b in terms of circuit moments from (40) and (41) in (39) we get the required final expression of BSM

Thus this is how we obtain the closed form slew metric formula. It can be seen that the derive slew metric expression is merely the function of first three moments of the impulse response of the Beta Distribution.

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### III.3 Extension for the ramp input

One might assume that the input waveform is a ramp input with slope \( T \), as shown in Fig. 2(a) [14]. The PDF of this waveform is a uniform distribution with mean \( \mu(I) = T/2 \) and

\[ \text{BSM} = T_{HI} - T_{LO} \]

From (30) and (31) we get the expression of BSM as

\[ \text{BSM} = [0.9pQ(p, q)]^p - [0.1pQ(p, q)]^p \]

### III.3.2 Extended Slew Metric for Ramp input

Since we need to verify the effectiveness of our model, we have extracted 208 routed nets containing 1026 sinks from an industrial ASIC design in 0.18 \( \mu \)m technology. It has to be noticed that we need to choose the nets in such a way so that the maximum sink delay is at least 10 ps and the delay ratio between closet and furthest sinks in the net is less than 0.2. It specifically ensures that each one of the net has at least one near end sink. We categorize and classify the 2244 sinks as it was taken in PERI [11] into the following three categories:

A number of 1187 far-end sinks have delay greater or equal to 75% of the maximum delay to the furthest sink in the net.

The output slew is the root-mean square of the step slew and input slew [14]. For ramp slew

\[ \text{slew}(R) = \sqrt{\text{slew}(s)^2 + \text{slew}(I)^2} \]

Further, Equation (43) exhibits the right limiting behaviour: as

\[ \text{slew}(I) \to \infty, \text{ we have } \text{slew}(R) \to \infty \text{ and as} \]
Slew(I) \to 0\), we have Slew(R) \to Slew(S). Where Slew(S) is the step slew metric which is given by equation (37) and
\[
0 = 2\left[\frac{i_1(t)-i_2(t)}{s}\right] + \frac{3}{2}\left[\frac{i_2(t)-i_2(0^-)}{s}\right] + \frac{3}{2}\left[\frac{i_1(t)-i_1(0^-)}{s}\right] + \frac{1}{2}\left[\frac{i_1(t)}{s} - \frac{i_2(t)}{s}\right] \quad ..........
\]
Slew(I) is the input slew which is given as

To find the ramp output slew at node 5, we use a saturate ramp of T=100 ps. For both estimation and calculation for slew, the relative error obtained is less than 2 %.

\[
\text{Slew}^2 (I) = \frac{T^2}{12}
\]

From equations (42), (47) and (48), we get

Next step is that we compare the obtained average, minimum, and also the maximum values along with standard deviation for slew from PERI with those found using our proposed model. The comparative results are summarized in Table 1.

\[
\text{Slew}(R) = \frac{m}{\text{e}^{1/s}} \cdot \frac{T}{12} \quad \text{mm}
\]

The above extended equation shown above (49) is the delay metric equation for the Beta Distribution function for the input signal which is a ramp input.

IV. EXPERIMENTAL RESULTTS

When we come to the experimental results we, by notifying that we have incorporated and implemented the proposed delay estimation method specifically using Gamma Distribution and applied it to widely used actual interconnect

![RLC Circuit](image)

Applying KVL in mesh 1
\[
V(t) = 3\frac{di(t)}{dt} + 2[i_1(t) - i_2(t)] + \int i_1(t) - i_2(t) dt
\]
Taking laplace transform
\[
0 = 2\left[\frac{i_1(s)-i_2(s)}{s}\right] + \frac{3}{2}\left[\frac{i_1(s)-i_1(0^-)}{s}\right] + \frac{3}{2}\left[\frac{i_2(s)-i_2(0^-)}{s}\right] + \frac{1}{2}\left[\frac{i_1(t)}{s} - \frac{i_2(t)}{s}\right] \quad ..........
\]

Taking laplace transform
\[
0 = i_2(s) \left[ -1 + \frac{1}{s} + 4s - \frac{1}{2s} \right] - i_1(s) \left[ 2 - \frac{1}{s} \right] + i_3(s) \left[ 3 + \frac{1}{2s} \right]
\]

Applying KVL in mesh 3
\[
0 = 3\left[i_1(t) - i_2(t)\right] + \frac{1}{2} \int i_3(t) - i_2(t) + 5 \frac{di_3(t)}{dt} + 4i_1(t) + \frac{1}{3} \int i_3(t)
\]

Taking laplace transform of the above equation
\[
0 = 2\left[i_1(s) - i_2(s)\right] + \frac{3}{2}\left[i_3(s) - i_1(0^-)\right] + 4i_1(s) + \frac{1}{3}i_3(s)
\]

By solving above equations we get the values of current
\[i_1(s), i_2(s)\text{ and } i_3(s)\]

We get the desired values of current.

For the final step for experiments, we have calculated the general effects of input slew on the delay and also the slew estimation for the same seven node RLC network as shown in the figure 3. Specifically we have chosen 10/90 input slew values of 200 and 500 picoseconds and delays and slews are precisely calculated for each node. Also we have made the comparison by comparing the result of our proposed model with PERI and RICE [11] are shown in the table 3 and table4 for delay and slew respectively.

V. CONCLUSION

We finally conclude by presenting an extended proposed efficient and precise interconnect delay metric for high speed VLSI designs from two inputs such as the ramp input and the step input. We have used Gamma probability distribution function to derive our metric. Our model has Elmore delay which acts as upper bound but with significantly less error. The novelty of our approach is justified by the calculating and making the comparison with that of the results obtained by using a simulator known as the spice simulator and hence using SPICE simulations.

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