Car Dynamics using Quarter Model and Passive Suspension; Part V: Frequency Response Considering Driver-seat

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ABSTRACT
The objective of the paper is to investigate the dynamics of a quarter-car model with passive suspension and considering the driver-seat. The model is a 3 DOF one excited by the irregularities of the road pavement. The model dynamics are studied in terms of the frequency response of the driver using its transmissibility. The paper presents new technical terms used to assess the suspension parameters which are the accumulated and mean transmissibility. The driver frequency response is studied for road pavement of irregularities having harmonic components of frequencies up to 10 Hz. The natural frequencies of the system are calculated and the driver transmissibility is evaluated for suspension parameters in the range 2.85 to 11.4 kN/m for stiffness and 0.5 to 8 kNs/m for damping coefficient.

Keywords: Car dynamics; quarter car model; passive suspension; frequency response, driver transmissibility.

I. INTRODUCTION

The study of car dynamics has special importance since it helps car designers to produce vehicles capable of achieving ride comfort to passengers according to international standards. On the other hand, such studies help authorities dealing with the requirement of speed control in residential districts or for security reasons to design speed humps properly without harming the vehicles or passengers. Frequency response is one of the dynamic means of investigating car dynamics.

Papagiannakis and Gajaranthi (1995) proposed a pavement roughness statistics based on the vertical sprung acceleration. They simulated the sprung mass acceleration using a quarter-car model. They showed that the sprung mass transfer function had a distinct sensitivity to a pavement roughness frequency of 3.5 Hz [1]. Sun (2001) presented a theoretical model for determining roughness index. He obtained the frequency response of quarter-car model for frequencies up to 23 Hz [2]. Lauwerys, Swevers and Sas (2002) discussed the development of a controller for a passenger car using active shock absorber through varying the damping characteristics with controlled valves. They tested the proposed controller using a quarter-car test rig. They demonstrated the frequency response of the system for frequencies up to 30 Hz [3]. Zuo and Nayfeh (2003) considered road random excitations as input to an 8 DOF full-car model using passive suspensions. They considered the optimization of passive suspensions as structure-constrained LQG/H2 optimal control problem. They demonstrated the vehicle frequency response for exciting frequencies up to 100 Hz [4].

Lu (2004) studied using the frequency-adaptive multi-objective suspension strategy for vehicle suspension control. He used a quarter-car model and presented the frequency response of the system for exciting frequencies up to 20 Hz [5]. Melcer (2006) used a quarter-car model in the solution of some problems of vehicle-road interaction in the frequency domain. He calculated the frequency response in terms of amplitude and phase with frequencies up to 100 rad/s [6]. Batterbee and Sims (2007) used skyhook damping laws within primary automotive suspension. They used a 2 DOF quarter-car model excited by realistic road profiles. They estimated the frequency response of the system using exciting frequencies up to 15 Hz [7]. Chi, He and
Naterer (2008) applied three optimization algorithms for the design optimization of vehicle suspensions based on a quarter-car model. The investigated the effect of car speed and road irregularity on the design variables to improve ride quality. They covered frequencies up to 48 Hz in the frequency response of the vehicle [8]. Santos et. al. (2010) modeled a MR damper as a component in a quarter-car model. The presented the frequency response of the sprung mass for frequencies up to 20 Hz [9].

Barbosa (2011) used a quarter-car model to identify the vehicle dynamic behavior. He derived the vehicle inertance function using the vehicle force transfer function. He obtained the vehicle frequency response for frequencies up to 100 Hz [10]. Slaski (2012) used signal processing to estimate the power spectral density and the magnitude characteristics of a car suspension. He used a quarter-car model subjected to input excitation with frequencies up to 25 Hz [11]. Patil, Pawar and Patil (2013) reviewed the research work regarding vehicle's passive and active suspensions. They presented the vehicle frequency response for exciting frequencies up to 100 Hz showing resonance at frequencies less than or equal 10 Hz [12]. Razdan, Bhave and Awasare (2014) studied the active pneumatic suspension with control strategy based on mass flow control for a small car. They concluded that the active suspension using velocity feedback provided less transmissibility at resonance compared to the passive suspension. They used frequencies up to 25 Hz in their presentation of the car frequency response [13]. Saglam and Unlusoy (2014) developed an active hydro-pneumatic suspension system. They used a quarter-car model and derived a nonlinear model with state dependent matrices. They presented the frequency response of the system for passive and active suspensions for frequencies up to 20 Hz. Their system showed resonance near 1 and 9 Hz [14]. Hassaan and Mohammed (2015) used a 10 DOF full-car model to investigate the frequency response of the vehicle against road disturbance. Their frequency response covered frequencies up to 18 Hz [15].

II. THE QUARTER-CAR MODEL

The quarter car model is used extensively in studying the vehicle dynamics. Large number of researchers consider this model as a 2 DOF one by considering only the tire-wheel assembly and the sprung mass. Here in this work I consider it as a 3 DOF by considering the driver and his seat. The line diagram of the 3 DOF quarter-car model is shown in Fig.1.

![3 DOF quarter-car model](image)

Figure 1 3 DOF quarter-car model.

The model of Fig.1 consists of:

- One tire and wheel and its attachments having the parameters: mass \( (m_1) \), damping coefficient \( (c_1) \) and stiffness \( (k_1) \).
- Sprung mass which is one quarter of the car chassis having the parameters: mass \( (m_2) \), damping coefficient \( (c_2) \) and stiffness \( (k_2) \). \( c_2 \) and \( k_2 \) are the suspension parameters.
- The driver and its seat having the parameters: mass \( (m_3) \), damping coefficient \( (c_3) \) and stiffness \( (k_3) \). \( c_3 \) and \( k_3 \) represent the seat damping and elasticity.

III. MODEL PARAMETERS

The 3 DOF quarter-car model parameters are borrowed from more than one reference. The parameters of the tire and sprung mass are used by Florin, Ioan-Cosmin and Liliana [16], while the parameters of the driver and seat are
used by Guclu [17]. The 3 DOF quarter-car model parameters used in this analysis work are given in Table 1.

### Table 1 3 DOF model parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_1$ (kN/m)</td>
<td>Tire stiffness</td>
<td>135</td>
</tr>
<tr>
<td>$c_1$ (kN/m)</td>
<td>Tire damping coefficient</td>
<td>1.4</td>
</tr>
<tr>
<td>$m_1$ (kg)</td>
<td>Un-sprung mass</td>
<td>49.8</td>
</tr>
<tr>
<td>$k_2$ (kN/m)</td>
<td>Suspension stiffness</td>
<td>2.85, 5.7, 11.4</td>
</tr>
<tr>
<td>$m_2$ (kg)</td>
<td>Sprung mass</td>
<td>466.5</td>
</tr>
<tr>
<td>$k_3$ (kN/m)</td>
<td>Seat stiffness</td>
<td>15</td>
</tr>
<tr>
<td>$c_3$ (Ns/m)</td>
<td>Seat damping coefficient</td>
<td>150</td>
</tr>
<tr>
<td>$m_3$ (kg)</td>
<td>Driver-seat mass</td>
<td>90</td>
</tr>
</tbody>
</table>

The 3 DOF model parameters

The damping coefficient of the suspension, $c_2$ is changed in the range:

$$0.5 \leq c_2 \leq 8 \text{ kNs/m}$$

The purpose of this change is investigate its effect of the frequency response of the sprung mass due to the irregularities of the road.

### IV. MATHEMATICAL MODEL

The mathematical model of the 3 DOF quarter-car model is defined by three differential equations obtained by applying Newton’s third law to the free body diagram of the three lumped masses of Fig.1. The dynamic motions of the system are: $x_1$, $x_2$ and $x_3$. The irregularities of the road is defined by the dynamic motion $y$ which is assumed sinusoidal. Of course, the irregularities are random. The frequency contents of this irregularity can be defined using FFT (fast Fourier transform). Thus, the sinusoidal input motion $y$ represents one component in this transform. All the parameters of the system are passive and assumed having the constant values presented in section 3. The differential equations are written in standard form as follows:

For the tire and wheel mass, $m_1$:

$$m_1x_1'' + (c_1+c_2)x_1' - c_2x_2' + (k_1+k_2)x_1 - k_2x_2 = c_1y' + k_1y$$

For the sprung mass, $m_2$:

$$m_2x_2'' - c_2x_1' + (c_2+c_3)x_2' - c_3x_3' - k_2x_1 + (k_2+k_3)x_2 - k_3x_3 = 0$$

For the driver and seat mass, $m_3$:

$$m_3x_3'' - c_3x_2' + c_3x_3' - k_3x_2 + k_3x_3 = 0$$

Equations 1, 2 and 3 can be written in the matrix form:

$$\mathbf{M}\mathbf{x}'' + \mathbf{C}\mathbf{x}' + \mathbf{K}\mathbf{x} = \mathbf{F}$$

Where: $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the system mass, damping and stiffness matrices of the 3 DOF system given respectively by:

$$\mathbf{M} = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} c_1+c_2 & -c_2 & 0 \\ -c_2 & c_2+c_3 & -c_3 \\ 0 & -c_3 & c_3 \end{bmatrix}$$

$$\mathbf{K} = \begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix}$$

The response vector $\mathbf{x}$ is:

$$\mathbf{x} = [x_1 \ x_2 \ x_3]^T$$

The force vector $\mathbf{F}$ in Eq.4 is:

$$\mathbf{F} = [F_1 \ 0 \ 0]^T$$

Where $F_1$ is the force acting on the tire and wheel mass as transmitted from the road through the viscoelastic characteristics of the tire. That is:

$$F_1 = c_1y' + k_1y$$

For a harmonic exciting force component of angular frequency $\omega$, $y$ takes the form:

$$y = Y e^{j\omega t}$$

Combining Eqs.7 and 8 gives:

$$F_1 = Y(k_1 + jc_1\omega) e^{j\omega t}$$
Where $Y$ is the peak amplitude of the harmonic component of the road irregularity.

The steady state solution of Eq.4 is:

$$x = X e^{jωt}$$

(10)

Where $X$ is the phasor of the amplitude vector of the three motions.

Now, combining Eqs.4 and 10 gives the amplitude phasor $X$ as:

$$X = (K - \omega^2 M + jωC)^{-1} F$$

(11)

Solving Eq.11 for any frequency range provides the frequency response for the three masses of the system.

It is possible to study the frequency response of the quarter-car model in terms of the transmissibility. This can be done as follows:

- The transmissibility here is a displacement transmissibility defined as $X/Y$ [18].
- The phasor of the transmissibility $TR$ is obtained by combining Eqs.6,9 and 11 as:

$$TR = (K - \omega^2 M + jωC)^{-1} [k_1 + jc_1,0,0]^T$$

(12)

- The transmissibility of any of the three masses will be the magnitude of its corresponding phasor.
- In the present study, the transmissibility of the driver is investigated.

V. QUARTER-CAR MODEL NATURAL FREQUENCIES

The evaluation of the natural frequencies of any dynamic system is of great importance since they assign the location of resonance during harmonic excitation of the system. The analyzed system has three degrees of freedom. Then, it has three natural frequencies. They are obtained using the equation:

$$(K - \omega_n^2 M)X = 0$$

(13)

The MATLAB command 'eig' is used to extract the system natural frequencies using Eq.13 as an eigenvalue problem [19]. The system natural frequencies depend on the suspension stiffness as given in Table 2.

Table 2 Quarter-car model natural frequencies

<table>
<thead>
<tr>
<th>Suspension stiffness, $k_2$ (kN/m)</th>
<th>2.85</th>
<th>5.70</th>
<th>11.40</th>
</tr>
</thead>
<tbody>
<tr>
<td>First natural frequency (Hz)</td>
<td>0.3555</td>
<td>0.4964</td>
<td>0.6847</td>
</tr>
<tr>
<td>Second natural frequency (Hz)</td>
<td>2.2497</td>
<td>2.2553</td>
<td>2.2666</td>
</tr>
<tr>
<td>Third natural frequency (Hz)</td>
<td>8.3737</td>
<td>8.4604</td>
<td>8.6322</td>
</tr>
</tbody>
</table>

VI. DRIVER TRANSMISSIBILITY

The driver displacement transmissibility provides the driver motion amplitude relative to the ground motion amplitude at a specific frequency. The isolators have to maintain this value less than unit value for proper suspension design. This depends mainly on the parameters of the suspension elements and the exciting frequency. Figs.2, 3 and 4 show the frequency response of the driver as function of the three parameters, exciting frequency $f$, damping coefficient $c_2$ and stiffness $k_2$. The three figures are generated by MATLAB using Eq.12 and the parameters in Table 1.

![Figure 2](image-url)
Figs. 2, 3 and 4 depict the driver transmissibility for frequencies between 0 and 10 Hs, suspension damping coefficient between 1 and 5 kNs/m and suspension stiffness between 2.85 and 11.40 kN/m.

Two resonance values appear in the three transmissibility plots which are the first and second resonance.

The third resonance does not appear.

All the transmissibility curves intersect at 2 points at zero frequency and at a frequency between the first and second resonance.

- After the second intersection the damping effect is reversed. That is decreasing the damping coefficient will decrease the transmissibility which this process is reversed between the first and second intersections.

VII. DRIVER MAXIMUM TRANSMISSIBILITY

- The driver transmissibility is maximum near resonance.
- It may be maximum near the first or second resonance as clear in Figs.2-4.
- The location of the maximum transmissibility depends on the suspension parameters $c_2$ and $k_2$.
- Fig.5 gives the maximum transmissibility of the driver against $c_2$ and $k_2$ (transmission parameters).
VIII. ACCUMULATED DRIVER TRANSMISSIBILITY

- This is a new function introduced by the author to assess the effectiveness of the transmission elements for the whole range under study (0 ≤ f ≤ f_{max}).
- The accumulated transmissibility, TR_{acc} is defined as the area under the transmissibility curve. That is:
  \[ TR_{acc} = \int_{0}^{f_{max}} TR \, df \] (14)
- MATLAB is used to evaluate the accumulated transmissibility of the driver through using its command 'sum' [20].
- The variation of the accumulated transmissibility with the suspension parameters is shown in Fig.6 as generated by MATLAB.

IX. MEAN DRIVER TRANSMISSIBILITY

- The driver transmissibility increases from a unit value to a maximum value, then decreases (or decreases, increases and decreases) again to almost zero value.
- Then, the mean value is used by the author as another way for assessing the effect of the suspension parameters c_2 and k_2.
- The mean is simply evaluated using the MATLAB command 'mean' [20].
- Fig.7 shows the variation of the transmissibility mean of the driver with the suspension parameters c_2 and k_2.

X. MINIMUM EXCITING FREQUENCY FOR TRANSMISSIBILITY < 1

- In suspension design, it is desired to produce driver transmissibility < 1 to attenuate the road
disturbance.
- This can be achieved by controlling the road pavement such that its irregularities do not have frequency components less than specific values.
- This is the minimum disturbance frequency, \( f_{\text{min}} \).
- Again \( f_{\text{min}} \) depends on the suspension parameters \( c_2 \) and \( k_2 \) as shown in Fig. 8.

\[
\begin{align*}
\text{Figure 8 Minimum frequency for transmissibility < 1.}
\end{align*}
\]

- For high level of damping coefficient (\( c_2 > 2 \) kN\( \cdot \)s/m) the suspension stiffness almost has no effect on the minimum frequency and \( f_{\text{min}} \) varies almost linearly with \( c_2 \).
- For \( c_2 < 2 \) kN\( \cdot \)s/m, \( f_{\text{min}} \) decreases as \( k_2 \) decreases at a specific frequency.
- It is possible to go down with \( f_{\text{min}} \) to \(< 0.5 \) Hz at small levels of the suspension parameters (\( c_2 \) and \( k_2 \)).

XI. CONCLUSION

Including the driver-seat in the quarter-car model has increased the degree of freedom (DOF) to 3. This resulted in three natural frequencies, i.e. three resonance locations with possible increase of the frequency response near those values. Two new parameters were introduced to the terminology of frequency response of dynamic systems which were the accumulated transmissibility and mean transmissibility. The main conclusions of this research work are as follows:

1. The dynamic model of the 3 DOF dynamic system was derived assuming passive parameters for all dampers and springs in the system.
2. The frequency response of the quarter-car model was presented in terms of the driver transmissibility.
3. New terms were introduced to assess the driver vibrations due to road disturbance (accumulated and mean transmissibility).
4. The three natural frequencies of the system were evaluated for different values of the suspension damping coefficient and stiffness.
5. Using MATLAB, the frequency response of the driver in terms of its transmissibility was presented for harmonic road frequencies up to 10 Hz, suspension parameters of damping coefficient in the range 0.5 to 8 kN\( \cdot \)s/m and stiffness in the range 2.85 to 11.4 kN/m.
6. The third natural frequency did not show any resonance effect in the frequency response of the driver.
7. The effect of the suspension parameters on the maximum driver transmissibility, accumulated transmissibility, mean transmissibility and minimum road frequency for transmissibility less than one was demonstrated.
8. Decreasing the values of the suspension parameters decreased both accumulated and mean transmissibility's.
9. The maximum transmissibility was greater than one for all the combinations of the suspension parameters.
10. It was possible to go down by the minimum road frequency to less than 0.5 Hz with suspension stiffness of 2.85 kN/m and damping coefficient \( \leq 1 \) kN\( \cdot \)s/m.

REFERENCES


BIOGRAPHY

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- Emeritus Professor of System Dynamics and Mechanical Vibrations.
- Has got his Ph.D. in 1979 from Bradford University, UK under the supervision of Late Prof. John Parnaby.
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