Tuning of a Novel Third-order Feedforward Compensator; Part I: Used with Underdamped Second-order-like Process

Galal Ali Hassaan
Department of Mechanical Design & Production, Faculty of Engineering, Cairo University, Egypt
Email: galalhassaan@ymail.com

ABSTRACT
The paper presents a novel third-order feedforward compensator to control underdamped second-order-like processes. Damping ratios of 0.05 and 0.20 of the second-order-like process are considered. The proposed compensator has four parameters to be tuned to adjust the performance parameters of the closed-loop control system. The MATLAB optimization toolbox is used to solve the resulting constrained optimization problem during compensator tuning. The use of ITAE, ISE, IAE, ITSE and ISTSE objective functions is investigated. Two functional constraints are used to limit the maximum percentage overshoot and the steady-state error of the closed-loop control system. Using the proposed compensator it is possible to go down with the maximum percentage overshoot from 85.4 to 0.35 % and the settling time from 5.97 to 2.10 s for a second-order-like process of 0.05 damping ratio with ITAE objective function. Very low level steady-state error is possible using the proposed compensator.

Keywords – Third-order feedforward compensator, underdamped second-order-like processes, control system performance, compensator tuning.

I. INTRODUCTION
Highly oscillating processes represent a challenge to control engineers. The reason for this is simply the desired objective of reducing the oscillation amplitudes of the closed loop control system to minimum and to make the system as fast as possible in terms of its settling time. One of the techniques suggested by the author is using a feedforward third-order compensator. The paper studied the validity of using the feedforward compensator in conjunction with an underdamped second-order-like process.

Feced, Zervas and Muriel (1999) presented a method for the design of complex fiber Bragg gratings. They designed second-order and third-order dispersion compensators [1]. Kajita, Moon and Temes (2000) introduced an architecture for sensor interface circuit using a delta-sigma modulator. They used a third-order delta-sigma structure to shape the operational amplifier noise [2]. Boujelben, Rebei, Dallet and Marchegay (2001) presented several filter topologies suited for data conversion and measurement applications. They used a sharpened comb filter of third-order [3].

Kuntman, Cicekoglu and Ozcan (2002) described current-mode third-order Butterworth filter topologies with unity gain active elements and minimum number of passive components. They used equal values passive capacitors and resistors [4]. Aksoy, Ozcan, Cicekoglu and Kuntman (2003) presented eight current-mode third-order band pass filter topologies employing unity-gain active elements, three capacitors and three resistors [5]. Janocha, Pesotski and Kuhnen (2008) developed a method for compensating complex hysteretic actuator and sensor characteristics. They used a third-order Butterworth low-pass filter having one parameter (cutoff frequency) [6].

Casson and Villegas (2010) investigated the use of standard filter approximations for Butterworth, Chebyshev and Bessel as an alternative wavelet approximation technique [7]. Lee and Manzie (2012) proposed a technique for the attenuation of brake judder at the source. They described an adaptive compensator to establish the brake torque variation and to produce a compensating clamp force. They used first-order, second-order and third-order compensators [8]. Gan, Todd and Apsley (2013) examine the impact of time delays on emulation systems. They used first-order and third-order compensators to compensate the time delay [9].

II. PROCESS

The process used in the investigation of using the feedforward third-order compensator is a second-order one having the following transfer function, \( G_p(s) \):

\[
G_p(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}
\]  

(1)

Where:

- \( \omega_n \) = process natural frequency (10 rad/s).
- \( \zeta \) = process damping ratio (0.05, 0.2).

The unit step response of the process is shown in Fig.1 for the three damping ratio values.

![Fig.1 Process time response to a unit step input.](image)

The time based specifications of the process for the three damping levels are given in TABLE 1.

<table>
<thead>
<tr>
<th>Damping ratio</th>
<th>Maximum overshoot (%)</th>
<th>Settling time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>85.4161</td>
<td>5.9749</td>
</tr>
<tr>
<td>0.20</td>
<td>52.5701</td>
<td>1.3740</td>
</tr>
</tbody>
</table>

III. COMPENSATOR

The compensator is a third-order compensator of constant coefficients having a transfer function, \( G_c(s) \) given by:

\[
G_c(s) = \frac{K_c}{s^3 + a_1 s^2 + a_2 s + a_3}
\]  

(2)

It has four parameters: \( K_c \), \( a_1 \), \( a_2 \) and \( a_3 \). The compensator gain \( K_c \) is included to improve the steady-state characteristics of the closed-loop control system comprising the feedforward compensator and the second-order process. The other three parameters help in adjusting the performance parameters of the closed-loop control system such as the maximum percentage overshoot and the settling time.

IV. CONTROL SYSTEM TRANSFER FUNCTION

The closed-loop block diagram of the control system consists of two cascaded blocks in the forward path for the compensator [with \( G_c(s) \)] and process [with \( G_p(s) \)]. The feedback elements have unit gain, i.e. the control system is a unity feedback one. Then, the transfer function of the system, \( M(s) \) is given by:

\[
M(s) = \frac{G_c(s)G_p(s)}{1 + G_c(s)G_p(s)}
\]  

(3)

Combining Eqs.1,2 and 3 gives the closed loop transfer function as:

\[
M(s) = \frac{K_c\omega_n^2}{s^5 + b_1 s^4 + b_2 s^3 + b_3 s^2 + b_4 s + b_5 + K_c\omega_n^2}
\]  

(4)

Where:

- \( b_1 = a_1 + 2\zeta\omega_n \)
- \( b_2 = a_2 + 2\zeta\omega_n a_1 + \omega_n^2 \)
- \( b_3 = a_3 + 2\zeta\omega_n a_2 + \omega_n^2 a_1 \)
- \( b_4 = 2\zeta\omega_n a_3 + \omega_n^2 a_2 \)
- \( b_5 = \omega_n^2 a_3 \)

V. CONTROL SYSTEM PERFORMANCE

The control system performance in the time domain is measured through [13,14]:

- Maximum percentage overshoot: For a computer-aided work, it is assigned using the MATLAB command ‘Stepinfo’ for a ±5% of the steady state value of the time response of the closed-loop control system [15].
- Settling time for a ±5% band of the steady-state response of the closed-loop control system.
- Steady state error of the control system defined as the difference between the step input amplitude and the steady-state response of the system (for unity feedback control systems).

For the system transfer function given by Eq.4, the steady-state error of the system, \( e_{ss} \) is:

\[
e_{ss} = \frac{a_3}{s^3 + K_c}
\]  

(5)

VI. TUNING THE THIRD-ORDER COMPENSATOR

The third-order compensator has four parameters to be adjusted to produce a satisfactory performance of the closed-loop control system. The tuning procedure is as follows:
1. The constrained optimization technique of MATLAB is applied through using its command ‘fmincon’ [16].
2. Two functional constrains are assigned to the maximum percentage overshoot and the steady state-error.
3. Compensator parameters are constrained as follows:
   - \(0.01 \leq K_c \leq 200\)
   - \(0.01 \leq a_1 \leq 500\)
   - \(0.01 \leq a_2 \leq 500\)
   - \(0.001 \leq a_3 \leq 2\)
4. The compensator parameters are tuned through minimizing an error-based objective function (ITAE, ISE, IAE, ITSE and ISTSE) [17-21] subjected to the functional and compensator parameters constraints.
5. The compensator tuning results are given in TABLES 2-4 for a process damping ratio of 0.05, 0.2 and 0.8 respectively.

**Table 2 Compensator tuned parameters and control system performance measure for \(\zeta = 0.05\).**

<table>
<thead>
<tr>
<th>Objective function</th>
<th>ITAE</th>
<th>ISE</th>
<th>IAE</th>
<th>ITSE</th>
<th>ISTSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>(K_c)</td>
<td>10.6195</td>
<td>26.2400</td>
<td>1.5224</td>
<td>-</td>
<td>85.3916</td>
</tr>
<tr>
<td>(a_1)</td>
<td>5.3471</td>
<td>500</td>
<td>87.9286</td>
<td>-</td>
<td>48.6731</td>
</tr>
<tr>
<td>(a_2)</td>
<td>12.0938</td>
<td>189.4374</td>
<td>39.9132</td>
<td>-</td>
<td>48.6731</td>
</tr>
<tr>
<td>(a_3)</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>-</td>
<td>0.0031</td>
</tr>
<tr>
<td>(OS_{\text{max}}) (%)</td>
<td>0.3497</td>
<td>1</td>
<td>0</td>
<td>-</td>
<td>1.0044</td>
</tr>
<tr>
<td>(T_s) (s)</td>
<td>2.0830</td>
<td>15.4367</td>
<td>73.7814</td>
<td>-</td>
<td>1.3748</td>
</tr>
<tr>
<td>(10^5 e_{\text{ss}})</td>
<td>9.4157</td>
<td>38.1080</td>
<td>65.6430</td>
<td>-</td>
<td>3.6026</td>
</tr>
</tbody>
</table>

The optimization technique with ITSE objective function did not converge to a global minimum.

**Table 3 Compensator tuned parameters and control system performance measure for \(\zeta = 0.20\).**

<table>
<thead>
<tr>
<th>Objective function</th>
<th>ITAE</th>
<th>ISE</th>
<th>IAE</th>
<th>ITSE</th>
<th>ISTSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>(K_c)</td>
<td>15.3171</td>
<td>0.3778</td>
<td>7.8728</td>
<td>200</td>
<td>22.9932</td>
</tr>
<tr>
<td>(a_1)</td>
<td>13.3070</td>
<td>107.116</td>
<td>499.615</td>
<td>10.539</td>
<td>7.2198</td>
</tr>
<tr>
<td>(a_2)</td>
<td>23.9766</td>
<td>10.500</td>
<td>103.888</td>
<td>121.32</td>
<td>21.9643</td>
</tr>
<tr>
<td>(a_3)</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.1948</td>
</tr>
<tr>
<td>(OS_{\text{max}}) (%)</td>
<td>0.7615</td>
<td>0.9980</td>
<td>1</td>
<td>0.9998</td>
<td>0</td>
</tr>
<tr>
<td>(T_s) (s)</td>
<td>3.2237</td>
<td>59.493</td>
<td>28.202</td>
<td>1.9061</td>
<td>1.7660</td>
</tr>
<tr>
<td>(10^5 e_{\text{ss}})</td>
<td>6.5282</td>
<td>264</td>
<td>12.7</td>
<td>0.50</td>
<td>840</td>
</tr>
</tbody>
</table>

**VIII. CONCLUSION**

- A novel third-order feedforward compensator was proposed to control underdamped second-order-like processes.
- The compensator had four parameters which were tuned using MATLAB optimization toolbox.
- The controlled second-order-like process had two levels of damping ratio (0.05 and 0.20).
- The process without any control had a step response of 85.41, 52.57 % maximum percentage overshoot and 5.79, 1.37 s settling time corresponding to the two damping levels.
- The proposed third-order compensator having four parameters was tuned using the MATLAB optimization toolbox.
- Five objected functions based on the error in the control system step response were used to tune the compensator (ITAE, ISE, IAE, ITSE and ISTSE).
- Using the proposed third-order compensator to control a second-order-like process of 0.05 damping ratio has reduced the maximum percentage overshoot of the closed-loop control system to only 0.35 % and the settling time to 2.10 when an ITAE objective function was used.
- Using the proposed third-order compensator to control a second-order-like process of 0.20 damping ratio has reduced the maximum percentage overshoot of the closed-loop control system to 0 % (no overshoot condition) and the settling time to 1.76 s when an ISTSE objective function was used.
- The steady-state error of the closed-loop control system using the proposed third-order compensator was as less as 65 x 10⁻⁷ with a second-order-like process with 0.05 damping ratio and 850 x 10⁻⁵ with 0.20 damping ratio process.
- The proposed third-order compensator has proven to be very efficient in generating a good-performance closed-loop control system when used with second-order-like processes having very low damping ratio.
- The proper selection of the optimization objective function had a clear impact in tuning the third-order compensator used with underdamped second-order-like processes.

REFERENCES

**BIOGRAPHY**

**Galal Ali Hassaan**
- Emeritus Professor of System Dynamics and Automatic Control.
- Has got his Ph.D. in 1979 from Bradford University, UK under the supervision of Late Prof. John Parnaby.
- Now with the Faculty of Engineering, Cairo University, EGYPT.
- Research on Automatic Control, Mechanical Vibrations, Mechanism Synthesis and History of Mechanical Engineering.
- Published 10’s of research papers in international journals and conferences.
- Author of books on Experimental Systems Control, Experimental Vibrations and Evolution of Mechanical Engineering.
- Chief Justice of IJCT.
- Reviewer in some international journals.
- Scholars interested in the authors publications can visit: http://scholar.cu.edu.eg/galal