Laser Induced Electron Acceleration in an Electromagnetic Standing Wave Wiggler with External Magnetic Tapering Parameters

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Abstract
In this paper the acceleration of electrons by a laser pulse, in an electromagnetic standing wave wiggler with an external tapered magnetic in vacuum has been investigated. The vector potentials of the laser pulse and magnetic wiggler and tapering external magnetic field are taken as $A_1, A_w, B_r$ respectively, we also have $\alpha$ as a tapering parameter. By applying an external magnetic tapering parameter, the trajectory of electrons in the x-y plane is changed from closed circular to a swinging in (y) direction. Besides, the axial velocity of electrons in propagation direction decreases and the fluctuation of velocity decrease with increasing of tapering parameters. It should be noted that by increasing of tapering parameters, the electron enhanced in axial direction, so that the energy gain will increase.

Keywords—Inverse Free Electron Laser, Accelerator, Wiggler, Tapered, Standing Wave

I. INTRODUCTION
In 1972 when Palmer first described the inverse free-electron lasers (IFELs)[1]. He shed light on future of high-gradient particle accelerators [2]. The IFEL is a promising stageable laser accelerator of electrons or positrons, along with possible gradients in the 100-400 Mev/m range[3], based on present technology. In the past decades several initial specimen of this field have been operated. In 1992, I. Wernick and T. C. Marshal first prepared an experiment that proposed IFEL acceleration which was followed by experiments using nanosecond-duration, GW, CO2 laser [4]. At the Columbia university, the same laboratory that the prior experiment was constructed in there, staging of two laser-driven, relativistic electron accelerators has been demonstrated for the first time in a proof-of-principle experiment, whereby two distinct and serial laser accelerators acted on an electron beam in a coherently cumulative manner [5]. (stella.pdf). In fact, the first IFEL serves to bunch the electrons into micro-bunches, which are further accelerated by the laser in the second IFEL. In 2003, the inverse Cerencov accelerator (ICA) has been shown [6]. A radially polarized laser beam which is focused by an axicon onto the e-beam traveling through a gas-filled interaction region [7]. There are also experimental proofs of the energy gain of a preaccelerated electron beam from a micro-wave inverse free electron laser accelerator (MIFELLA) [8]. The inverse free electron laser accelerator (IFELA) is an instrument that is served by IFEL (Inverse Free Electron Laser) as a particle (electrons, positrons, etc.) accelerator. The IFELA’s interaction occurs in vacuum and away from boundaries. That’s why it’s a desirable device for accelerating particles. It resolves the problems of using the plasma as an accelerating medium (including plasma instabilities, nonlinear laser propagation, shot-to-shot reproducibility, and the extremely small accelerating potential well) which disorder plasma-based accelerator plans. To obtain good focusing, electron-beam optics and transport, the inverse free electron laser wiggler is appropriate. It provides the high-quality electron beam that is required for advanced light sources, biomedical applications, and the Next Linear Collider. However, the basic IFELA plan suffers from the phase-slippage of the confined electron on the analogy of the drive laser wave. As the electron obtains energy, we can’t control the free-electron laser (FEL) resonance condition [9], and therefore the electron gains a maximum energy mandatorily given by the FEL interaction bandwidth. This problem can be troubleshooted by either using a tapered wiggler period and/or amplitude [10,11] or, in parallel, by using a chirped drive laser pulse. Hartmannin 2001 tested the chirped-pulse inverse free-electron and showed that by using a femtosecond (fs), ultra-high-intensity drive laser pulse, the IFEL interaction bandwidth and accelerating gradient were increased, hence large energy gains produced.

II. RELATIVISTIC ANALYSIS
Suppose the emission of laser pulse beam is assumed with the following vector potential:

$$A_l (z, t) = -A_0 \sin(\alpha t - kz) \exp\left(\frac{-t}{\tau_g} \right) (\frac{kz}{c} \frac{\omega^2}{\omega_p^2})$$

(1)

Where $z_0$ is the initial position of the pulse peak, $k = (\frac{\omega}{c})(1 - \frac{\omega_p^2}{\omega^2})^{\frac{1}{2}}$, group velocity $v_g = c (1 - \frac{\omega_p^2}{\omega^2})^{\frac{1}{2}}$, $\tau_g$ is the pulse duration, $L_j$ is the wiggler period and/or amplitude. It should be noted that $L_j$ can be expressed by the FEL interaction bandwidth $\Delta \omega_j$.

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plasma frequency \( \omega_p = (4\pi n_e e^2 / m)^{1/2} \), and \(-e\) and \(m\) are the electronic charge and effective mass, respectively. According to Maxwell equations, the Electric field \( \mathbf{E} \) and magnetic field \( \mathbf{B} \) are connected to potentials vector in the form of:

\[
\mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{A}}{\partial t}, \quad \mathbf{B} = \nabla \times \mathbf{A} \tag{2}
\]

These equations are defined to Corresponding vector potential and we used them for electric field \( E_L \), magnetic field \( B_L \) of laser beam, Wiggler electric field \( E_w \), Wiggler magnetic field \( B_w \), external beam emitter magnetic field \( B_e \), and corresponding electric field \( E_r \).

To investigate the equation of energy and momentum of accelerated electrons and interaction of this beam with electrons, we numerically study the trajectory of electron that accelerated by laser in electromagnetic standing wave wiggler in presence of external magnetic beam emitter parameters.

First we introduce the equation governing electric and magnetic fields and corresponding vector potential for laser beam, wiggler field and external tapering parameters.

\[
B_w (x,t) = -B_{aw} \cos(k_w x) \sin(\alpha_w t) \dot{z} \tag{3}
\]

\[
B_e (z) = B_{ae} (1 + \alpha z) \dot{z} \tag{4}
\]

\[
A_w (x,t) = -A_{aw} \sin(k_w x) \sin(\alpha_w t) \dot{y} \tag{5}
\]

\[
A_e (z) = B_{ae} z (1 + \frac{\alpha z}{2}) \dot{y} \tag{6}
\]

Where \( k_w \) is the wave vector, \( \omega_w \) wiggler angular frequency and \( \alpha \) is wiggler field and external tapering parameter.

By considering relativistic electron energy and equations of electric and magnetic fields, the equations of energy and momentum of electron are:

\[
A_x = A_L (z,t) \dot{x}, \quad A_y = A_w (x,t) + A_r (z), \quad A_z = 0
\]

\[
\frac{dp}{dt} = -e[E + (V \times B)] \tag{7}
\]

\[
\frac{dp_x}{dt} = -e \frac{\partial A_L}{\partial t} + (p_y) \frac{\partial A_L}{\partial z} = -e \frac{d}{dt} (A_x) = -e \frac{d}{dt} (A_L) \tag{8}
\]

\[
\frac{dp_y}{dt} = -e \frac{\partial A_w}{\partial t} + (p_y) \frac{\partial A_w}{\partial z} = -e \frac{d}{dt} (A_y) = -e \frac{d}{dt} (A_w + A_r) \tag{9}
\]

\[
\frac{dp_z}{dt} = -e \frac{\partial A_e}{\partial t} + (p_y) \frac{\partial A_e}{\partial z} = -e (\dot{y}) = -e (\dot{y}) \frac{\partial A_e}{\partial z} \tag{10}
\]

\[
\frac{dy}{dt} = e \frac{1}{m_e} \frac{\partial B_e}{\partial z} \tag{11}
\]

With direct integration of (8), (9), Electrons momentum in \( x \) and \( y \) direction yields:

\[
P_x = a_L + c_1, \quad P_y = a_w + c_2 \tag{12}
\]

In equation (12):

\[
a_L = -a_{w0} \sin(\alpha t - k z) \exp[-(t - (z - z_L) / \gamma_L^2 / \tau_L^2)] \tag{13}
\]

\[
a_w (x,t) = -a_{w0} \sin(k_w x) \sin(\alpha_w t), a_r (z) = a_{r0} z (1 + \frac{\alpha z}{2}) \tag{14}
\]

While \( a_{w0} = A_{w0} e / m_c, a_{w} = B_{aw} e / m_c, a_{r} = A_{aw} e / m_c \).

If we consider \( p_x = 0 \) and \( p_y = 0 \) in starting point \((t=0)\), the upper coefficients yield \( c_1 = c_2 = 0 \). By using relativistic relations \( \gamma^2 = 1 + (p_x^2 + p_y^2 + p_z^2) / m_c^2 \), and equation (12), \( p_z \) take the form:

\[
P_z = \frac{1}{m_c} \left[ \gamma^2 - 1 - (a_L^2 + (a_w + a_r)^2) \right] \tag{15}
\]

In the following discussion we will use the dimensionless form of equations (12) and (15), then the trajectory and energy equation given by:

\[
\frac{dx}{dt} = k (a_L) \tag{16}
\]

\[
\frac{dy}{dt} = k (a_w + a_r) \tag{17}
\]

\[
\frac{dz}{dt} = k \left[ \gamma^2 - 1 - (a_L^2 + (a_w + a_r)^2) \right] \tag{18}
\]

\[
\frac{d\gamma}{dt} = a_{0L} \frac{dx}{dt} + a_{0w} \frac{dy}{dt} + a_{0r} \frac{dz}{dt} \tag{19}
\]

Trajectory and energy Equations are coupled ordinary differential equations. We solve them numerically by fourth order Runge–Kutta method and plot electron trajectories in \( x-y, \dot{z} - z \) and \( \gamma - z \). In this paper, we study the trajectory of relativistic electrons in \( x-y \) plane in the presence of a tapered magnetic wiggler. We also study Electron velocity in propagation direction and investigate of tapered magnetic wiggler on it. We present our result as dimensionless variables involved magnetic field, tapering parameters, wave number, laser beam period and wiggler wave number period.

III. NUMERICAL RESULTS
To analyze the electron trajectory in $x-y$ plane, we assume that the initial momentum is 0 then we study the effect of tapering parameters on it when:

$$a_{0L} = 200, a_{w0} = 20, k = 100, a_{0t} = 0.1$$

Figure 1. Trajectory of electron in $x-y$ plane when $\alpha = 0.6$

In Fig. 1, we show tapering parameter in electron trajectory in $x-y$ plane ($E_{xy}$) that changes as beamer parameters change. Without tapering parameters (fig.1), $E_{xy}$ is closed circular that make possible to determine the electron limited area. By applying the tapering parameter, $E_{xy}$ is changed. When the tapering parameter increases, electron leaves closed circular trajectory and swings in ($y$) direction and progresses.

We study the velocity of electrons in propagation direction in the presence of a tapered parameter with:

$$a_{0L} = 200, a_{w0} = 20, k = 100$$ and $a_{0t} = 0.1$

Figure 2. Effect of laser wave number parameter on the accelerated electron energy.

Figure 2 shows that with increasing wave-number, as it was expected, the energy of the electron increases. Also, axial velocity of electrons in propagation decreases during a time interval. The tapered parameter influences the changing of velocity and decrease the fluctuation of velocity when increases. Omitting of tapered parameter increases the resonance intensity of velocity and the distance of fluctuation.

IV. DISCUSSION

The trajectory of accelerated electron by a laser pulse in $x-y$ plane by applying an external magnetic tapering wigglers is strongly affected by the change of external tapering parameter. The electrons leave their closed circular trajectory and start to swing in ($y$) direction when tapering parameter increases. The increase of this parameter from zero decrease variation of axial velocity graph and it changes from falling high fluctuated curve to falling high fluctuated one. This change affected the energy gain by electron and it reduces the fluctuation of its graph and enhanced the variation of energy gain.

REFERENCES