Differential Evolution Technique for the Design of High-Pass FIR Digital Filter

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ABSTRACT
This paper presents the robust design of Linear phase high-pass finite impulse response (FIR) digital filter using evolutionary technique called Differential Evolution (DE) and the impact of different mutation strategies have been studied based on minimum fitness function for the optimal design of FIR filter. DE is stochastic population based optimization algorithm which is used for multidimensional real valued functions and it explore the search space locally as well as globally to obtain the optimal design parameters. From the simulation results it has been shown that the applied algorithm DE can optimize the digital FIR filter in terms of minimizing the magnitude approximation error and ripple magnitudes of pass-band and stop band.

KeyWords---- Differential Evolution, Finite Impulse Response (FIR) digital filter, Magnitude Response.

I. INTRODUCTION

Today digital image processing (DSP) is used in wide variety of fields like communication, image processing, digital television, pattern recognition etc. Because of numerous advantages such as more efficient, more flexibility, environment stability, ease to store and use, lower equipment production cost than analog signal processing, it become an important application area in the field of engineering and need new advancements. The new advancements comes with advancements in digital filtering. Filters perform two operations, signal separation and signal restoration. Signal separation is done when signal is contaminated with noise, interference or other signal and signal restoration is done when signal is distorted in some way, thus filters improves the signal quality by boost or attenuate the signal. Both of these filtering operations are done by using analog and digital filters. Analog signals operate on continuous time signals and consists of active and passive components like capacitors, resistors, operational amplifiers etc. Digital filters consists of DSP processor and controller that perform mathematical operations on sampled, discrete time signals to reduce or enhance certain aspects of signal. In analog filters limitations exists due to electronic components such as stability and accuracy of resistors and capacitors whereas in digital filters these problems does not exist. Because of increased complexity the digitals filters may be more expensive than analog filters but many large practical designs are implemented with digital filters that are impossible with analog filters.

Digital filters are of two types FIR(Finite Impulse Response) and IIR(Infinite Impulse Response). In FIR filter, the impulse response is of finite duration which settles to zero within a finite interval of time and output of FIR filter depends on only present and past input samples. FIR filters have number of advantages over IIR filter. FIR filters have guaranteed stability because all the poles are present at the origin thus all are within the unit circle and FIR filters have linear phase response because of symmetric nature of coefficients and these are to be easily designed [3].

Traditionally, three well known FIR filter design methods are used such as window method, frequency sampling method and optimal filter design method. The windowing method consists of truncating or windowing a theoretically ideal filter impulse response by some suitably chosen window function. The window function is fast, simple because of presence of well defined equation and robust but it offer very little design flexibility and generally suboptimal. Various kinds of windows (Butterworth, Chebyshev, Kesar and Hamming) are used based on filter specifications such as transition width, ripples in pass-band and stop-band attenuation. The major disadvantage is lack of accurate control of pass and stop-band cut-off frequencies [5,7].

The objective function, for the optimal filter design required accurate control of various parameters and is highly non-differentiable, non-uniform and multi-modal in nature. In optimal method various methods exists in which filter coefficients are designed again and again until particular error is to be minimized. So evolutionary optimization methods are to be implemented for the design of optimal digital filter which are quite efficient for the design of FIR filter such as GA, stimulated annealing, artificial bee colony optimization, Tubu search, PSO, DE etc [11]. Genetic Algorithm (GA) is based on the principle of natural selection and genetics and has a capability of searching multidimensional and multimodal spaces. It is more efficient for locating the local optimum point but is not very successful in determining the global optima because sometimes it may trap to the local optima due to the presence of various
local optima of objective function in high dimensional problems and it also have a slow convergence speed. Kennedy and Ebehart developed the global search particle swarm optimization (PSO) algorithm which is based on the behavior of bird flocking. PSO is simple elastic, faster convergence, easy to implement and has robust search capability. Despite many advantages PSO have many disadvantages like if the initial parameters are not chosen correctly then results stuck into local minima which may result in divergent particle trajectories so global best position is not achieved and sometimes premature convergence occur in high dimensional problems [4,12].

This paper presents the evolutionary optimization technique of Differential Evolution for the design of digital high-pass FIR filter. The non-linear, non-differential objective function have many local minima, therefore to deal with these functions or to find a true global minima a powerful global search optimization algorithm is required. DE has proven successful for overcome all such problems. It is also to tackle the numerically complex computation problems. Differential Evolution differs from GA in terms of DE use mutation as the primary search mechanism whereas GA rely on crossover. DE was first introduced by Storn and Price in 1995. DE have many advantages comparable to other algorithms like DE require few control parameters, fast convergence and has the ability to find the the true global minima regardless of initial parameter values and providing multiple solutions in a single run [1,9].

The paper is arranged in four sections. Section II describes the differential evolutionary algorithm for designing the optimal high-pass FIR filter. Section IV describes the simulation results that have been achieved. Finally, section V concludes the paper.

II. FIR FILTER DESIGN PROBLEM

FIR filter using feed-forward difference equation because in this filter there is no feedback of past and future outputs to form the present value, thus operates in real time and depends only on past and present inputs to form the output. FIR filter specifications include minimizing the magnitude approximation errors and ripples in passband and stop-band. The difference equation is expressed as:

\[ y(n) = \sum_{k=0}^{M} h(n) x(n - k) \]  \hspace{1cm} (3)

The output \( y(n) \) is represented by the convolution of impulse response \( h(n) \) with the input \( x(n) \). Both \( h(n) \) and \( x(n) \) are of finite duration sequences, so their convolution is also finite in duration. The transfer function of FIR filter is given as:

\[ H(z) = \sum_{k=0}^{M} h(n) z^{-k} \]  \hspace{1cm} (4)

The even order linear phase high-pass FIR filter is designed for which only half of the coefficients are determined instead of all coefficients due to symmetric and anti-symmetric properties. For the symmetric and anti-symmetric FIR filters the condition for linear phase is:

\[ h(n) = h(M - n) \]  \hspace{1cm} (5)

In this paper only symmetric property is to be used, so \( \frac{M+1}{2} \) coefficients are to be determined for the design of filter.

The design condition for the high-pass FIR filter is:

\[ H_d(\omega_s) = \begin{cases} 
1 & \text{if } 0 \leq \omega_s \leq 0.7\pi \\
0 & \text{if } 0.8\pi \leq \omega_s \leq \pi 
\end{cases} \]  \hspace{1cm} (6)

\( s = 0 - 140 \) (for stop-band)
\( s = 160 - 200 \) (for pass-band)
\( \omega_s = \frac{2\pi}{k} s = 0, 1, ..., k \)

k = 200 samples

The performance of digital FIR filter can be calculated by using \( L_1 \)-norm and \( L_2 \)-norm approximation error of magnitude response and ripple magnitude of both pass-band and stop-band. The FIR filter has been designed by optimizing the coefficients in such a way that magnitude approximation errors and ripple magnitudes are to be minimized.

\[ err_1(x) = \sum_{i=0}^{k} |H_{\text{ideal}}(\omega_s) - |H(\omega_s, x)|| \]  \hspace{1cm} (7)

\[ err_2(x) = \sum_{i=0}^{k} \left( |H_{\text{ideal}}(\omega_s) - |H(\omega_s, x)|| \right)^2 \]  \hspace{1cm} (8)

\( err_1(x) \)- absolute error \( L_1 \)- norm of magnitude response
\( err_2(x) \)- squared error \( L_2 \)-norm of magnitude response

Desired magnitude response \( H_{\text{ideal}}(\omega_s) \) of FIR filter is defines as:

\[ H_{\text{ideal}}(\omega_s) = \begin{cases} 
1, & \text{for } \omega_s \in \text{passband} \\
0, & \text{for } \omega_s \in \text{stopband} 
\end{cases} \]  \hspace{1cm} (9)

The ripple magnitudes of pass-band and stop-band are to be minimized which are given by \( \delta_p(x) \) and \( \delta_s(x) \) respectively.

\[ \delta_p(x) = \max(|H(\omega_s(x))| - \min(|H(\omega_s(x))|) \]  \hspace{1cm} (10)

\[ \delta_s(x) = \max(|H(\omega_s, x)|) \]  \hspace{1cm} (11)

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Aggregating all objectives, the multi-criterion optimization problem is stated as:

Minimize $O_1(x) = err_1(x)$

Minimize $O_2(x) = err_2(x)$

Minimize $O_3(x) = \delta_p(x)$

Minimize $O_4(x) = \delta_s(x)$

A single optimal tradeoff point can be calculated by converting multi-objective function into single objective function as following:

Minimize $O(x) = \sum_{j=1}^{4} \omega_j O_j(x)$ (12)

III. DIFFERENTIAL EVOLUTION FOR FIR FILTER DESIGN

DE is heuristic population based robust direct search method which is used for optimize real valued non-linear, non-differentiable continuous functions, proposed by Storn and Price in 1995. DE includes the basic steps of initialization, mutation, crossover and selection to evaluate the final resultant solution from a randomly generated population. The variants used for DE/α/β/γ. DE stands for Differential Evolution, α represents the vector to be perturbed, β represents the number of difference vectors that will used for perturbation, γ tells about crossover mechanism to create new offspring.

1. Parameter Setup

Control parameters used in DE are: population size (S), mutation factor ($F_m$), crossover rate (CR), boundary constraints of optimization variables (NG), $P_{ij}$ is $j^{th}$ element of NG set of coefficients of filter giving $i^{th}$ element of population.

2. Initialization of Population

This indicates the starting of algorithm. Set generation $T=0$. Initialize population $P_{ij}$ with values which are randomly chosen in the feasible numerical range of space. Initialize the population by taking lower and upper limits of search space.

$$p_{ij}^T = p_{ij}^{min} + rand(0,1)(p_{ij}^{max} - p_{ij}^{min})$$ (j=1,2,...,NG; i=1,2,...,S) (13)

$rand(0,1)$ is uniformly distributed random number in the range between 0 and 1.

3. Mutation Operation

This operation makes the DE different from other evolutionary algorithms which applies the vector differentials between the randomly chosen population vectors and adds to the third one. Mutation begins by randomly chosen three population vectors. Five different mutation strategies have been applied which are defined as:

$$V_{ij}^T = p_{R_{1j}}^T + F_m(p_{R_{1j}}^T - p_{R_{2j}}^T)$$ (14)

$$V_{ij}^T = p_{bestj}^T + F_m(p_{bestj}^T - p_{R_{2j}}^T)$$ (15)

$$V_{ij}^T = p_{ij}^T + F_B(p_{bestj}^T - p_{ij}^T) + F_m(p_{R_{1j}}^T - p_{R_{2j}}^T)$$ (16)

$$V_{ij}^T = p_{bestj}^T + F_m(p_{bestj}^T + p_{R_{2j}}^T - p_{R_{1j}}^T - p_{R_{2j}}^T)$$ (17)

$$V_{ij}^T = p_{R_{1j}}^T + F_m(p_{R_{1j}}^T + p_{R_{2j}}^T - p_{R_{1j}}^T - p_{R_{2j}}^T)$$ (18)

(i=1,2,...,S; j=1,2,...,NG)

4. Crossover Operation

In which the trial vector is generated from the elements of target and donor vector by replacement of the certain elements of target vector by the corresponding elements of donor vector. The trial vector is defined as:

$$u_{ij}^{T+1} = \begin{cases} V_{ij}^T & \text{if } (R_4(i) \leq C_r) \text{ or } (j = R_5(i)) \\ p_{ij}^T & \text{if } (R_4(j) > C_r) \text{ or } (j \neq R_5(i)) \end{cases}$$ (j = 1, 2, ..., NG; i = 1, 2, ..., S) (19)

Where $u_{ij}^{T+1} = [u_{i1}^{T+1}, u_{i2}^{T+1}, ..., u_{iNG}^{T+1}]^T$ is the trial vector, $C_r$ is the crossover rate in the range between [0,1], $V_{ij}^T$ is donor vector, $p_{ij}^T$ is target vector, $R_4(j)$ is the $j^{th}$ evaluation of a uniform random number generation with [0,1]. $R_5(i) \in \{1, 2, ..., NG\}$ is randomly chosen using a uniform distribution.

5. Selection Operation

In which the values of obtained new individual is compared with the original individual to decide whether or not the obtained new individual will be the member of next generation.

$$p_{ij}^{T+1} = \begin{cases} u_{ij}^{T+1} (j = 1, 2, ..., NG) & \text{if } f(u_{ij}^{T+1}) < f(p_{ij}^T) \\ p_{ij}^T (j = 1, 2, ..., NG) & \text{otherwise} \end{cases}$$ (i=1,2,...,S) (20)

The objective function of each trial vector $u_{ij}^{T+1}$ is compared with that of target vector $p_{ij}^T$. If the objective function of trial vector is less than target vector, then the value of trial vector is assigned for next generation otherwise target vector replaces the trial vector.

6. Verification of Stopping Criterion

Generation number is set at $T+1$. All the process is repeated until stopping criterion met, usually the maximum number of iterations $T_{max}$. Mainly the stopping criterion depends on the problem type.
7. Algorithm

FIR filter design using differential evolution algorithm:
1. Read data i.e. population size, mutation factor, crossover rate and maximum number of iterations.
2. Generate an array of uniform random numbers.
3. Generate initial population individual and compute augmented objective function.
4. Arrange calculated objective function in ascending order and select first half of the population members.
5. Set iteration counter, \( T = 0 \).
6. Increment iteration counter, \( T = T + 1 \).
7. Select best member.
8. Generate an array of uniform random numbers and generate five different integer random numbers then applied mutation operation using differential mutation strategies. Compute augmented objective function and find mutant vector based on minimum augmented objective function.
9. Generate arrays of random numbers and apply crossover and selection methods.
10. Is stopping criteria met.
11. No – Go to 6
12. Write best result
13. Stop.

IV. RESULTS AND DISCUSSIONS

The high-pass FIR filter is designed by using DE algorithm, five different mutation strategies have been explored at different orders and one best mutation strategy is chosen on the basis of minimum objective function achieved and then parameter tuning has been performed. To design digital FIR high-pass filter values are analysed at equally spaced 200 samples.

1. Analysis of Objective Function at Different Orders

For DE, mutation strategy-2 has been applied by varying filter order from 20 to 50. Objective function values are calculated corresponding to each filter order as shown in Figure 1.

![Fig.1 Filter Order vs Objective Function](image)

Fig.1 shows that the objective function is decreasing as the filter order value increases. The filter order 20 shows the maximum value of objective function and filter order 50 shows the minimum value of objective function. So performance comparison of results at order 20 and at order 50 of digital high pass FIR filter has been carried out.

2. Objective Function vs Mutation Strategy at Filter Order 20 and 50

Different five mutation strategies have been applied at filter order 20 and 50 and the corresponding values of achieved objective function have been given as shown in Table 1.

Table 1. Objective Function at Different Mutation Strategies

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Mutation Strategy</th>
<th>Objective function for filter order 20</th>
<th>Objective function for filter order 50</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mutation-strategy 1</td>
<td>5.520988</td>
<td>0.468602</td>
</tr>
<tr>
<td>2</td>
<td>Mutation-strategy 2</td>
<td>5.520990</td>
<td>0.483087</td>
</tr>
<tr>
<td>3</td>
<td>Mutation-strategy 3</td>
<td>5.520934</td>
<td>0.459423</td>
</tr>
<tr>
<td>4</td>
<td>Mutation-strategy 4</td>
<td>5.520945</td>
<td>0.460999</td>
</tr>
<tr>
<td>5</td>
<td>Mutation-strategy 5</td>
<td>5.520994</td>
<td>0.482493</td>
</tr>
</tbody>
</table>

Hence the Table 1 shows that for filter order 20 mutation strategy-2 gives the best result and for filter order 50 mutation strategy-3 gives the best results.

3. DE Parameter Tuning

Firstly value of population has been varied from 50 to 150 in steps of 10 and obtained values of objective function at each order have been shown in Table 2.

Table 2: Objective Function Values at Different Population

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Population Size</th>
<th>Objective function for filter order 20</th>
<th>Objective function for filter order 50</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>5.520936</td>
<td>0.495835</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
<td>5.520895</td>
<td>0.487869</td>
</tr>
<tr>
<td>3</td>
<td>70</td>
<td>5.520926</td>
<td>0.492043</td>
</tr>
<tr>
<td>4</td>
<td>80</td>
<td>5.520934</td>
<td>0.491170</td>
</tr>
</tbody>
</table>
The Table 2 indicates that at filter order 20, Population value 60 and at filter order 50, population value 100 gives the best value of objective function. Corresponding to filter order 20 and 50 the variation in the value of achieved objective function are shown in Fig 2 and Fig 3 respectively.

Now the Value of Mutation Factor ($F_m$) have been varied by keeping the population size fixed at 60 and 100 for filter order 20 and 50 respectively. The value of $F_m$ has been varied from 0.6 to 1.0 in steps of 0.05. The achieved values of objective function with respect to variations in mutation factor have been shown in Table 3.

Table 3: Mutation Factor vs Objective Function

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Mutation Factor</th>
<th>Objective Function at Filter Order 20</th>
<th>Objective Function at Filter Order 50</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.60</td>
<td>5.520993</td>
<td>0.481235</td>
</tr>
<tr>
<td>2</td>
<td>0.65</td>
<td>5.520995</td>
<td>0.506133</td>
</tr>
<tr>
<td>3</td>
<td>0.70</td>
<td>5.520965</td>
<td>0.501585</td>
</tr>
<tr>
<td>4</td>
<td>0.75</td>
<td>5.520981</td>
<td>0.508896</td>
</tr>
<tr>
<td>5</td>
<td>0.80</td>
<td>5.520895</td>
<td>0.459423</td>
</tr>
<tr>
<td>6</td>
<td>0.85</td>
<td>5.520923</td>
<td>0.490311</td>
</tr>
<tr>
<td>7</td>
<td>0.90</td>
<td>5.520971</td>
<td>0.518194</td>
</tr>
<tr>
<td>8</td>
<td>0.95</td>
<td>5.520977</td>
<td>0.520152</td>
</tr>
<tr>
<td>9</td>
<td>1.00</td>
<td>5.523541</td>
<td>2.151179</td>
</tr>
</tbody>
</table>

Hence the Table 3 depicts that minimum value of objective function is achieved for filter order 20 and 50 respectively when mutation factor ($F_m$) is taken as 0.8. The variations of objective function are shown in Fig. 4 and Fig. 5 corresponding to filter order 20 and 50.
function is observed when \( F_m \) is taken 0.8 and objective function increases rapidly beyond 0.95. Now value of crossover rate have been varied by keeping \( F_m \) value fixed at 0.8 for both filter orders with population value fixed at 60 and 100 for filter order 20 and 50 respectively.

Table 4: Crossover Rate vs Objective Function

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Crossover Rate</th>
<th>Objective Function at Filter Order 20</th>
<th>Objective Function at Filter Order 50</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.10</td>
<td>5.520951</td>
<td>0.473986</td>
</tr>
<tr>
<td>2</td>
<td>0.15</td>
<td>5.520962</td>
<td>0.481680</td>
</tr>
<tr>
<td>3</td>
<td>0.20</td>
<td>5.520895</td>
<td>0.459423</td>
</tr>
<tr>
<td>4</td>
<td>0.25</td>
<td>5.520919</td>
<td>0.485361</td>
</tr>
<tr>
<td>5</td>
<td>0.30</td>
<td>5.520895</td>
<td>0.482868</td>
</tr>
<tr>
<td>6</td>
<td>0.35</td>
<td>5.520898</td>
<td>0.496348</td>
</tr>
<tr>
<td>7</td>
<td>0.40</td>
<td>5.520893</td>
<td>0.491671</td>
</tr>
</tbody>
</table>

Fig. 6 Crossover Rate vs Objective Function at Filter Order 20

Fig. 7 Crossover Rate vs Objective Function at Filter Order 50

The Fig. 6 and Fig. 7 depict the minimum value of CR is 0.4 and 0.2 for filter order 20 and 50 respectively.

Hence the results depict that at filter order 20 with mutation strategy-2, population size 60, \( F_m \) 0.8 and CR 0.4 and at filter order 50 mutation strategy-3, population size 100, \( F_m \) 0.8 and CR 0.2 gives the optimum value of objective function for the implemented DE algorithm.

Table 6: Maximum, Minimum, Average Value of objective function and Standard Deviation

<table>
<thead>
<tr>
<th>Filter Order</th>
<th>Maximum Value of Objective Function</th>
<th>Minimum Value of Objective Function</th>
<th>Average Value</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>5.520893</td>
<td>5.521925</td>
<td>5.521111</td>
<td>0.000153</td>
</tr>
<tr>
<td>50</td>
<td>0.679527</td>
<td>0.459423</td>
<td>0.542483</td>
<td>0.03599</td>
</tr>
</tbody>
</table>

Hence the Table 6 indicates that value of standard deviation is less than one for both at filter order 20 and 50 which shows the robustness of designed filter.

4. Magnitude and Phase Response Analysis

Magnitude and Phase response have been studied for the filter order 20 and 50. They show the amplification and attenuation values in different frequency bands.

Fig. 8 Magnitude(dB) vs Normalized Frequency at Filter Order 20

Fig. 9 Magnitude(dB) vs Normalized Frequency at Filter Order 50
The Fig. 8 and Fig. 9 shows that the attenuation at filter order 20 is 18.00145 dB and at filter order 50 is 43.83983 dB which shows that at filter order 50 the stop band attenuation is more.

![Phase vs Normalized Frequency for order 20 using mutation strategy-2](image1.png)

**Fig. 10** Phase vs Normalized Frequency for order 20 using mutation strategy-2

![Phase vs Normalized Frequency for order 50 using mutation strategy-3](image2.png)

**Fig. 11** Phase vs Normalized Frequency for order 50 using mutation strategy-3

Fig.10 and Fig.11 depict that the phase is linear in pass-band and stop-band.

![Magnitude vs Normalized Frequency for filter order 20 using mutation strategy-2](image3.png)

**Fig.12** Magnitude vs Normalized Frequency for filter order 20 using mutation strategy-2

![Magnitude vs Normalized Frequency for filter order 50 using mutation strategy-3](image4.png)

**Fig.13** Magnitude vs Normalized Frequency for filter order 50 using mutation strategy-3

**V. CONCLUSION**

Using DE algorithm the high-pass FIR filter has been designed at order 20 and 50. Five different mutation strategies have been applied at filter order 20 and 50. Out of these five mutation strategies one mutation strategy is to be selected based on minimum objective function. At filter order 20, the mutation strategy-2 and at filter order 50, the mutation strategy-3 gives the best objective function. By keeping the mutation strategy-2 and mutation strategy-3 fixed at filter order 20 and 50 the value of population size, mutation factor and crossover rate is varied to get the desired value of objective function. At filter order 20 the population size 60, \(F_m\) value 0.8, CR value 0.4 and at filter order 50 the population size 100, \(F_m\) value 0.8, CR value 0.2 gives the best value of objective function but at filter order 50 the objective function value is less than at filter order 20. The magnitude plots depict that at filter order 50 more stop band attenuation is present. Hence if we ignore the delay the filter order 50 gives the optimum value of objective function. The standard deviation at both filter orders is less than one which shows the robustness of filters.

**REFERENCES**


