Design of Digital Band Pass FIR Filter Using Heuristic Optimization Technique

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ABSTRACT
This paper presents the efficient way of designing Finite Impulse Response (FIR) band pass filter using Differential Evolution algorithm. In this various strategies of differential evolution have been attempted for the designing of FIR band pass filter. Impulse response coefficients of designed FIR filters have been represented as the sum or differences of powers of two individuals. The performance of filters depends upon its magnitude response and its hardware cost. DE is the heuristic approach which helps in minimizing possibly non-linear and non-differentiable functions. This method converges faster with more certainty. This includes initialization, mutation, crossover and selection. This also requires few control parameters like population size, mutation factor and crossover rate. DE is robust, easy to use but sometimes causes instability problems.

Keywords - Differential Evolution algorithm, FIR filters, Magnitude response, ripple magnitude.

I. INTRODUCTION
Digital signal processing has a wide range of applications like in communication fields, pattern recognition, image processing etc. It becomes more popular application in electronic engineering because of some advantages like more flexibility, good performance, better time response, environmental stability and lesser cost of equipment production. The main component of DSP is digital filters and it is defined as those are those systems on which mathematical operations are performed on sampled discrete time signals to boost some aspects of that signal. This is the filter which helps in performing all basic functions of DSP like filtering, addition of signals or separation of signals etc.

Processing of digital data is accomplished by wide varieties of digital filter and these are classified into two main categories: one is Infinite Impulse Response (IIR) Filters and other is Finite Impulse Response (FIR) Filters. FIR filters are those filters whose impulse response is of finite duration. These filters are non-recursive type filters because their output depends on present and past input values whereas IIR filters are those whose impulse response is of infinite duration and they are recursive type filters because their output depends on past output values. FIR filters has following advantages as compared to IIR filters: (1) finite impulse response (2) easy to optimize (3) linear phase (4) more stable.

Optimization is a procedure of finding and comparing various solutions until no better solution can be found. To design FIR band pass filter, various optimization techniques have been developed. Optimization techniques are divided into two categories: one is based on classical optimization and other is nature based optimization. Classical optimization includes direct search method and gradient method whereas nature based optimization includes Genetic Algorithm (GA), Particle Swarm Optimization (PSO) and Differential Evolution (DE). There are various drawbacks of gradient method such as function must be continuous, differentiable and completely defined and drawback of direct search method is that this method is valid for local optimum point because validation of global optimum is difficult. Hence nature based optimization is used because it is based on random variables.

GA is capable of searching multi-dimensional and multimodal decision spaces. This has also been used for the optimization of complex and discontinues functions. Sometimes GA attempts very low convergence and high dimensional optimization problems. Due to the existence of various local optima, GA traps in the local optima of objective function.

To overcome these disadvantages PSO has been developed by Kennedy and Ebenhart. PSO is simple, easy, fast and robust. It has some shortcomings also such as convergence behaviour which depends upon its parameters. If the parameters have been changed incorrectly which results in divergent particle trajectories which traps into local minima due to which convergence problem has been occurred which may lead to high dimensional optimization problem.
To overcome convergence property, DE has been developed by Storn and Price in 1995. DE is capable for finding global minimum regardless of the initial parameter values. It is simple and robust. It has few control parameters. It has parallel processing nature and fast convergence. DE has the ability to provide multiple solutions in a single run. DE is also capable of finding optimal solution for non-linear optimization problems.

The objective function for the design of digital filters involves various control parameters which are highly non-uniform, non-linear, non-differentiable, multimodal in nature. These objective functions cannot be optimized by classical optimization because of some disadvantages: (1) highly sensitive when no. of variables increase which leads to increase the size of space (2) frequent convergence to local optimal solution or divergence or revisiting the same sub optimal solution (3) requires continues and differentiable objective cost function (4) problem of convergence and complexity. To overcome these drawbacks, traditional and classical methods of optimization have been developed in which DE is one of them which is meta heuristic approach which helps in minimizing non-linear and non-differentiable functions and this has been developed by Storn and Price in 1995.

II. FIR FILTER DESIGN PROBLEMS

FIR filters are known as non-recursive type filters which are described by the difference equation expressed as:

\[ y(n) = \sum_{k=0}^{N-1} a_k x(n-k) \]  

Where \( a_k \) is the set of filter coefficients. The output \( y(n) \) is the function of only input signal \( x(n) \). \( N \) is the order of filter. FIR filter specifications include the maximum tolerable pass band ripple, maximum tolerable stop band ripple, pass band edge frequency and stop band edge frequency. The difference equation can be expanded as:

\[ y(n) = a_0 x(n) + \ldots + a_{N+1} x(n-N+1) \]  

The transfer function of FIR filter is given as:

\[ H(z) = \sum_{k=0}^{N-1} a_k z^{-k} \]  

The unit sample response of the FIR system is identical to the coefficients \( \{ a_k \} \), that is, \( h(n) = \left\{ \begin{array}{ll} a_n, & 0 \leq n \leq N-1 \\ 0, & \text{otherwise} \end{array} \right. \)

FIR filters have symmetric and anti-symmetric properties, which are related to their \( h(n) \) under symmetric and asymmetric conditions as described below by equations:

\[ h(n) = h(N-1-n) \quad \text{for even (symmetric)} \]  
\[ h(n) = -h(N-1-n) \quad \text{for odd (anti-symmetric)} \]

For such a system the number of multiplications is reduced from \( N \) to \( N/2 \) for \( N \) even and to \((N-1)/2 \) for odd. Then for linear phase FIR filter, the following condition should be satisfied:

\[ h(n) = \pm(N-1-n) \quad ; \quad n=0,1\ldots N-1 \]  

Errors: The FIR filter is designed by optimizing the coefficients in such a way that the approximation error function in \( L_p \) norm for magnitude is to be kept minimal. The magnitude response is specified at \( K \) equally spaced discrete frequency points in pass-band and stop-band.

\[
err_{1}(x) = \sum_{i=0}^{N} |H_d(\omega_i) - |H(\omega_i, x)| | \]  

\[
err_{2}(x) = \left( \sum_{i=0}^{N} (|H_d(\omega_i) - |H(\omega_i, x)|)^2 \right)^{1/2} \]  

where \( err_{1}(x) \) – absolute error \( L_1 \)-norm of magnitude response, \( err_{2}(x) \) – squared error \( L_2 \)-norm of magnitude response and \( N \) is number of samples.

Desired magnitude response \( H_d(w_i) \) of FIR filter is defined as:

\[ H_d(\omega_i) = \left\{ \begin{array}{ll} 1, & \text{for } \omega_i \in \text{passband} \\ 0, & \text{for } \omega_i \in \text{stopband} \end{array} \right. \]  

The ripple magnitudes of pass-band and stop-band have to be minimized which are given by \( \delta_p(x) \) and \( \delta_s(x) \) respectively.

\[ \delta_p(x) = \max \{|H(\omega_i, x)| - \min |H(\omega_i, x)|\} \]  
\[ \delta_s(x) = \max \{|H(\omega_i, x)|\} \]  

Aggregating all objectives, the multi-criterion constrained optimization problem is stated as:

Minimize \( O_1(x) = err_{1}(x) \)  
Minimize \( O_2(x) = err_{2}(x) \)  
Minimize \( O_3(x) = \delta_p(x) \)  
Minimize \( O_4(x) = \delta_s(x) \)

In multiple-criterion constrained optimization problem for the design of digital FIR filter, a single optimal design can be found by solving following equations:

Minimize \( O(x) = \sum_{i=1}^{4} \omega_i O_i(x) \)  

where \( \omega_i \) denotes the weights.

III. DIFFERENTIAL EVOLUTION

It is a population based stochastic method applied to minimize performance index. DE is the combination of arithmetic and classical operations of recombination, mutation and selection to evolve from a randomly generated starting population to a final solution. The different variants of DE are classified using: \( DE/alpha/beta/\delta \). \( \alpha \) indicates the method for selecting the parent chromosome that will form the base of the mutated vector. \( \beta \) indicates the number of difference vectors to perturb the base chromosome. \( \delta \) indicates recombination mechanism used to create offspring population. The bin acronym indicates that the recombination is controlled
by a series of independent binomial experiments. This is the method which is based on principles of GA’s but with crossover and mutation operations. The major difference is GA’s relies on crossover and DE relies on mutation. DE has few control parameters which made it popular. DE algorithm is classified into four steps: (1) initialization (2) mutation (3) crossover (4) selection and controlling parameters are (1) size of population (2) dimension (3) mutation factor (4) crossover rate.

1. Initialization
To initialize an individual population firstly set generation  \( g = 0 \) then initialize a population individuals with random values generated in search space according to the uniform probability distribution. This vector population is initialized within the upper and lower limits of the search space.

\[
p_{ij}^g = p_{ij}^{min} + \text{rand}(p_{ij}^{max} - p_{ij}^{min})
\]

where \( j = 1,2, ..., N_G; i = 1,2, ..., L \)

The population vector may violate constraints and these violations are corrected by fixing them either at lower or at upper limit.

2. Mutation Operation
Mutation is an operation that adds a vector differential to a population vector of individuals according to the following mutation strategies:

Various Differential mutation strategies:
There are several mutation strategies that can be employed for optimization.

Mutation Strategy 1:

\[
Y_{ij}^g = P_{Bj}^g + f_m(P_{ij}^g - P_{Bj}^g)
\]

where \( j = 1,2, ..., N_G; i = 1,2, ..., L \)

Mutation Strategy 2:

\[
Y_{ij}^g = P_{Bj}^g + f_m(P_{Bj}^g - P_{ij}^g) + f_m(P_{r1j}^g - P_{r2j}^g)
\]

where \( j = 1,2, ..., N_G; i = 1,2, ..., L \)

Mutation Strategy 3:

\[
Y_{ij}^g = P_{Bj}^g + f_m(P_{Bj}^g - P_{ij}^g) + f_m(p_{r2j}^g - P_{r2j}^g)
\]

where \( j = 1,2, ..., N_G; i = 1,2, ..., L \)

Mutation Strategy 4:

\[
Y_{ij}^g = P_{ij}^g + f_m(P_{Bj}^g + P_{ij}^g - P_{r1j}^g - P_{r2j}^g)
\]

where \( j = 1,2, ..., N_G; i = 1,2, ..., L \)

Mutation Strategy 5:

\[
Y_{ij}^g = P_{ij}^g + f_m(P_{r1j}^g + P_{r2j}^g - P_{r3j}^g - P_{r4j}^g)
\]

where \( j = 1,2, ..., N_G; i = 1,2, ..., L \)

The population vector may violate constraints and these violations are corrected by fixing them either at lower or at upper limit.

3. Crossover operation
Following the mutation operation, crossover is applied to the population. Crossover is employed to generate a trial vector by replacing certain parameters of the target vector by the corresponding parameters of a randomly generated donor vector.

For each vector, \( Y_{ij}^{g+1} \), an index \( r_5(i) \in \{1,2, ..., NG\} \) is randomly chosen using a uniform distribution, and a trial vector, \( U_{ij}^{g+1} = [u_{ij}^{g+1}, u_{i2}^{g+1}, ..., u_{iNG}^{g+1}]^T \)

\[
U_{ij}^{g+1} = \begin{cases} Y_{ij}^g & \text{if} \ (r_4(j) \leq CR) \ or \ (j = r_5(i)) \\ P_{ij}^g & \text{if} \ (r_4(j) > CR) \ or \ (j \neq r_5(i)) \end{cases}
\]

where \( j = 1,2, ..., N_G; i = 1,2, ..., L \)

The population vector may violate constraints and these violations are corrected by fixing them either at lower or at upper limit.

4. Selection Operation
Selection is the procedure whereby better offspring are produced. To decide whether the vector \( U_{ij}^{g+1} \) should be a member of the population comprising the next generation, it is compared with the corresponding vector \( P_{ij}^g \). Thus, if \( f \) denotes the cost function under minimization, then

\[
p_{ij}^{g+1} = \begin{cases} U_{ij}^{g+1} & \text{if} \ f(U_{ij}^{g+1}) < f(P_{ij}^g) \\ P_{ij}^g & \text{otherwise} \end{cases}
\]

where \( i = 1,2, ..., L; j = 1,2, ..., N_G \)

In this case, the cost of each trial vector \( U_{ij}^{g+1} \) is compared with that of its parent target vector \( P_{ij}^g \). If the cost function \( f \) of the target vector \( P_{ij}^g \) is lower than that of the trial vector, the target is allowed to advance to the next generation. Otherwise, a trial vector replaces the target vector in the next generation.

5. Control Parameters
The control parameters which controls DE that are population size, mutation factor, crossover rate and the stopping criteria (\( T_{\text{max}} \)).

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6. Algorithm for DE
Designing of FIR filter using DE algorithm:
The search procedure of the proposed differential evolution method has been discussed below:
1. Read data: population size, mutation factor, crossover rate, and maximum number of iterations.
2. Generate an array of uniform random numbers.
3. Generate the initial population individually and compute augmented objective function.
4. Arrange calculated objective function in ascending order and select first half of the population members.
5. Set iteration counter.
6. Increment the iteration counter.
7. Select best member.
8. Apply mutation, crossover and selection operations.
9. If \( g < T_{\text{max}} \) then go to 6.
10. Write GBEST.
11. Stop.

IV. DESIGN OF FIR FILTERS AND RESULTS:
The band-pass digital FIR filter has been designed using five different mutation strategies of Differential Evolution (DE). The DE algorithm has been implemented by varying the filter order from 12 to 34 and other DE parameters such as population size, mutation factor and crossover rate. The DE algorithm for mutation strategy 4 has been implemented for filter order from 12 to 34 and the filter order from achieved value of Objective Function has been decided with optimum value of Objective Function. Five mutation strategies given in Eq.19, Eq.20, Eq.21, Eq.22 and Eq.23 have been implemented on the selected filter order. Then the DE algorithm parameter such as population size, mutation factor and crossover rate have been varied on selected mutation strategy. The magnitude and phase response of designed digital FIR filters have been plotted. The results have been shown below:-

1. Selection of Order
The filter order has been varied from 12 to 34 using mutation strategy 4 of DE algorithm. The achieved Objective Function for each order is given in Table 1.

Table 1: Filter Order v/s Objective Function

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Filter Order</th>
<th>Objective Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>12</td>
<td>11.693170</td>
</tr>
<tr>
<td>2.</td>
<td>14</td>
<td>11.693200</td>
</tr>
<tr>
<td>3.</td>
<td>16</td>
<td>4.025982</td>
</tr>
<tr>
<td>4.</td>
<td>18</td>
<td>4.026045</td>
</tr>
<tr>
<td>5.</td>
<td>20</td>
<td>2.034361</td>
</tr>
<tr>
<td>6.</td>
<td>22</td>
<td>2.034530</td>
</tr>
<tr>
<td>7.</td>
<td>24</td>
<td>2.021790</td>
</tr>
<tr>
<td>8.</td>
<td>26</td>
<td>2.025935</td>
</tr>
<tr>
<td>9.</td>
<td>28</td>
<td>0.745541</td>
</tr>
<tr>
<td>10.</td>
<td>30</td>
<td>1.599873</td>
</tr>
<tr>
<td>11.</td>
<td>32</td>
<td>4.064672</td>
</tr>
<tr>
<td>12.</td>
<td>34</td>
<td>7.216461</td>
</tr>
</tbody>
</table>

From the Fig.1 it has been observed that objective function is minimum when filter order is 28. As filter order is increased beyond 28, the objective function increases rapidly. Order 28 exhibits the minimum value of objective function. Now all five mutation strategies have been applied at filter order 28.

2. Different Mutation Strategies for Order 28
As filter order 28 is selected now. Then different mutation strategies as described in Eq.19, Eq.20, Eq.21, Eq.22, Eq.23 are applied at filter order 28.

Table 2: Mutation Strategies v/s Objective Function

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Mutation Strategy</th>
<th>Objective Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Mutation Strategy 1</td>
<td>0.752657</td>
</tr>
<tr>
<td>2.</td>
<td>Mutation Strategy 2</td>
<td>1.374178</td>
</tr>
<tr>
<td>3.</td>
<td>Mutation Strategy 3</td>
<td>0.750166</td>
</tr>
<tr>
<td>4.</td>
<td>Mutation Strategy 4</td>
<td>0.745541</td>
</tr>
<tr>
<td>5.</td>
<td>Mutation Strategy 5</td>
<td>0.803539</td>
</tr>
</tbody>
</table>

Table 2 shows that mutation strategy 4 gives minimum objective function.
3. Selection of Population Size
As filter order 28 and mutation strategy 4 is selected. Now different population varying from 50 to 150 in the steps of 10 on mutation strategy 4 whose order is 28 and the results are given in Table 3.

Table 3: Population Size v/s Objective Function

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Population Size</th>
<th>Objective Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>50</td>
<td>0.754350</td>
</tr>
<tr>
<td>2.</td>
<td>60</td>
<td>0.756531</td>
</tr>
<tr>
<td>3.</td>
<td>70</td>
<td>0.753228</td>
</tr>
<tr>
<td>4.</td>
<td>80</td>
<td>0.745625</td>
</tr>
<tr>
<td>5.</td>
<td>90</td>
<td>0.749732</td>
</tr>
<tr>
<td>6.</td>
<td>100</td>
<td>0.745541</td>
</tr>
<tr>
<td>7.</td>
<td>110</td>
<td>0.744351</td>
</tr>
<tr>
<td>8.</td>
<td>120</td>
<td>0.744594</td>
</tr>
<tr>
<td>9.</td>
<td>130</td>
<td>0.744478</td>
</tr>
<tr>
<td>10.</td>
<td>140</td>
<td>0.744405</td>
</tr>
<tr>
<td>11.</td>
<td>150</td>
<td>0.744151</td>
</tr>
</tbody>
</table>

Table 3 shows that population 150 gives minimum objective function so that population 150 is selected and graph of population versus objective function is shown in Fig. 2.

4. Selection of Mutation Factor
As mutation strategy 4, filter order 28 and population 150 is selected. Keeping filter order as 28 and population size as 150, the mutation factor has been varied from 0.6 to 0.95 in the steps of 0.05 and the achieved value of objective function has been depicted in Table 4.

Table 4: Mutation Factor v/s Objective Function

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Mutation factor($f_m$)</th>
<th>Objective Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>0.60</td>
<td>0.742710</td>
</tr>
<tr>
<td>2.</td>
<td>0.65</td>
<td>0.769261</td>
</tr>
<tr>
<td>3.</td>
<td>0.70</td>
<td>0.748500</td>
</tr>
<tr>
<td>4.</td>
<td>0.75</td>
<td>0.774302</td>
</tr>
<tr>
<td>5.</td>
<td>0.80</td>
<td>0.744151</td>
</tr>
<tr>
<td>6.</td>
<td>0.85</td>
<td>0.746278</td>
</tr>
<tr>
<td>7.</td>
<td>0.90</td>
<td>0.745664</td>
</tr>
<tr>
<td>8.</td>
<td>0.95</td>
<td>0.753231</td>
</tr>
</tbody>
</table>

Table 4 shows that mutation factor 0.60 gives minimum objective function when applied on mutation strategy 4 whose order is 28 and population is 150 and the graph of mutation factor versus objective function is shown in fig. 3.
This graph shows that minimum objective function is obtained at mutation factor 0.60.

5. Selection of Crossover Rate (CR)
As mutation strategy 4, filter order 28, population 150, mutation factor 0.60 is selected. Keeping filter order as 28, population size as 150, mutation factor as 0.60, the crossover rate has been varied from 0.10 to 0.35 in the steps of 0.05 and the achieved value of objective function has been depicted in Table 5.

Table 5: Crossover Rate v/s Objective Function

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Crossover Rate (CR)</th>
<th>Objective Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.10</td>
<td>0.761768</td>
</tr>
<tr>
<td>2</td>
<td>0.15</td>
<td>0.762411</td>
</tr>
<tr>
<td>3</td>
<td>0.20</td>
<td>0.742710</td>
</tr>
<tr>
<td>4</td>
<td>0.25</td>
<td>0.743566</td>
</tr>
<tr>
<td>5</td>
<td>0.30</td>
<td>0.746786</td>
</tr>
<tr>
<td>6</td>
<td>0.35</td>
<td>0.743851</td>
</tr>
</tbody>
</table>

Table 5 shows that crossover rate 0.20 gives minimum value of objective function and the graph of crossover rate versus objective function is shown below in fig. 4.

Fig. 4: Crossover Rate v/s Objective Function

This graph shows that minimum objective function is achieved at crossover rate 0.20.

The chosen values of population, mutation factor, crossover rate are 150, 0.6, and 0.2 respectively. The maximum number of iteration has been taken 100 and executes 100 times to achieve minimum value of objective function. By combining these results we concluded that DE algorithm with filter order 28, mutation strategy 4, population size 150, mutation factor 0.6 and crossover rate 0.2 gives the optimum results for designing a band-pass digital FIR filter.

The magnitude plot shows the amplification and the attenuation in the different frequency bands.

Fig. 5: Magnitude v/s Normalized frequency for FIR designing filter with order 28 using mutation strategy 4

Fig. 6: Magnitude v/s Normalized Frequency for Band pass Digital FIR Filter

Fig. 7: Phase v/s Normalized Frequency for FIR designing filter with order 28 using mutation strategy 4

The range of pass-band and stop-band has been taken as $0.4\pi \leq \omega \leq 0.6\pi$ and $0 \leq \omega \leq 0.25\pi$, $0.75\pi \leq \omega \leq \pi$.

Table 6: minimum value, maximum value, average value and standard deviation of order 28
This table shows the minimum value, maximum value, average value and standard deviation of order 28.

V. CONCLUSION

In this thesis, filter order 28 has been selected for the digital band-pass FIR filter with 29 filter coefficients. Five different mutation strategies of DE have been applied at selected filter order 28. The mutation strategy 4 gives best objective function. After this, values of DE algorithm parameters have been varied for achieving the optimum value of objective function. It is concluded that population size 150, mutation factor 0.6, crossover rate value 0.20 gives the optimum value of objective function for the designed algorithm of digital band-pass FIR filter. The magnitude and phase responses also have been studied.

REFERENCES