Nature Inspired Optimization Technique used for Design of Band Stop FIR Digital Filter

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ABSTRACT

In this paper, band stop digital FIR filter is designed by using five different mutation strategies of differential evolution (DE) Algorithm. DE is population based stochastic optimization technique used to minimize the performance index. In Differential Evolution Algorithm the parameter has been tuned. In each step, the DE mutates vectors by adding random vector differentials to them. The one which is better the target or trial vector replaces the other in next iteration. Simulation results justify that by adjusting Population size, Mutation factor, Crossover rate the DE algorithm can minimize Magnitude Error and Ripple Magnitude Error in Stop Band and Pass band.

Keywords –Band stop, Differential Evolution (DE) Algorithm, FIR Digital Filters, Magnitude Error and Ripple Magnitude Error.

1. INTRODUCTION

Anything that contains information can be considered as signal. Signal can be identified into two types, continuous-time signals and discrete-time signals. A discrete-time signal is one that is defined at discrete instants of time. The numerical manipulation of signals and data in discrete time signals is called Digital Signal Processing (DSP). Digital Signal Processing presents greater flexibility, high attenuation and selectivity, better environment stability and lower equipment production costs than traditional analog techniques. DSP is one of the most powerful technology and most widely used. DSP has a wide range of applications in fields of communication, image processing, pattern recognition etc. These new DSP application result from advances in digital filtering. Digital Filter is a system based on Mathematical Operations which is applied on the sampled and discrete time signal to reduce or obtain certain aspects of that signal [4, 8, 19]. Filter eliminates the harmful constituent from the signal. Filter is filtering these noise signal and generate a noise less signal these noise less signal provide accurate information to the system. Filtering is a process by which frequency spectrum of a signal can be modified, re-shaped, or manipulated according to the desired specifications. The digital filter is a system in which both input and output are discrete time signals. There are two different types of different filters: finite impulse response (FIR) and infinite impulse response (IIR). An FIR filter is that whose impulse response is of finite duration. The output of such a filter is calculated from the current and previous input values. This type of filter is called non-recursive filter. An IIR filter is one whose impulse response continues for infinite duration. The output of IIR filter depends upon current and previous input and output values. This type of filter is called recursive filter. FIR filter has many advantages as compared to IIR filters such as: finite impulse response, easy to optimize, linear phase, better stability. Different fields where FIR filters are applicable are image processing, data transmission, signal processing etc. The filtering action can be determined by frequency response characteristic, which normally depends upon the choice of filter parameter i.e., the coefficients of the of the difference equations representing a filter in time-sample domain. If the coefficients of the equation are properly selected then frequency selective filters can be designed that will pass particular frequency components and rest all will be attenuated [4, 13, 20].

The objective function of filter design involves accurate control parameters of frequency spectrum and is thus highly non-uniform, non-linear, non differentiable and multimodal in nature. Classical optimization methods cannot coverage to solutions because they have disadvantages such as: highly sensitive to starting points, frequent convergence to local optimum solution or divergence or revisiting the same solution, requirement of continuous and differentiable objective cost function, requirement of the piecewise linear cost approximation and problem of convergence and algorithm.
Evolutionary optimization methods for design of optimal digital filters has better control parameters and highest stop band attenuation. Different methods have been developed; they are quite efficient for design of FIR filters like Genetic Algorithm, Stimulated Annealing, and Differential Evolution Algorithm [12].

This paper explores different mutation strategies of Differential Evolution algorithm optimization technique for design of digital band stop (BS) FIR filter. DE Algorithm was first introduced by Storn and Price in 1995 and has been successfully applied in the optimization of some well known non-linear, non-differentiable and non-convex functions. DE is greatly influenced by the choice of the mutation strategy, proper selection of appropriate mutation law plays an important role in searching the best results. DE has proved to be more accurate, reliable and can provide optimum solutions within acceptable computational times [4, 15].

The objective of this paper is to propose DE method for the design of band stop FIR digital filter that randomly explores the search space globally and locally. The values of the FIR filter coefficients are optimized with the differential Evolution to achieve magnitude error and ripple magnitude error of band stop as objective functions for optimization problem.

The paper is organized in five different sections as follows. The design formulation of FIR digital Filter is given in section 2. Section 3 discusses the Overview of Differential Evolution Algorithm design for FIR digital Filter, Section 4 consists of Simulation results obtained from Band stop (BS) FIR digital filter, Section 5 concludes the paper.

2. DESIGN FORMULATION OF DIGITAL FILTER

Digital Filter are classified in two broad categories which are named as Finite Impulse Response (FIR) and Infinite Impulse Response (IIR). This paper is focused to the design of digital Band Stop Filter. Filter design depends only on present and past inputs called Non – Recursive filter which means there is no feedback. The impulse Response of the FIR Filter is given by

\[ H(z) = h(0) + h(1)z^{-1} + \ldots + h(N)z^{-N} \] (1)

Or, \[ H(z) = \sum_{n=0}^{N} h(n)z^{-N} \] (2)

where, \( H(z) \) is the frequency domain representation of the Impulse Response and \( h(n) \) is the Impulse Response in time domain representation. The difference equation representation is described as

\[ y(n) = h(0)x(n) + h(1)x(n-1) + \ldots + h(N)x(n-N) \] (3)

Where order of the filter is \( N \), while the length of the filter is equal to the number of impulse response coefficients, i.e. \( (N+1) \). The impulse Response \( h(n) \) of the filter will help to determine filters type, for e.g. Low pass (LP), High pass (HP), Band pass (BP), Band stop (BS). This paper presents the filter to be even symmetrical Band stop filters. The main advantage of FIR filter is that it is always symmetrical, so the coefficients are also symmetrical. Due to the symmetrical nature dimension of the problem is halved and only half the coefficients are updated. The frequency Response of the FIR digital filter is given in equation below:

\[ H(\omega_k) = \sum_{n=0}^{N} h(n)e^{-j\omega_k n} \] (4)

Where \( H(\omega_k) \) is the Fourier transform complex factor and \( \omega_k = \frac{2\pi k}{N} \). This is the FIR filter frequency Response.

The frequency with \( N \) points in \((0, \pi)\)

\[ H_d(\omega) = \left[H_d(\omega_0), H_d(\omega_1), H_d(\omega_2), \ldots, H_d(\omega_N)\right]^T \] (5)

\[ H_i(\omega) = \left[H_i(\omega_0), H_i(\omega_1), H_i(\omega_2), \ldots, H_i(\omega_N)\right]^T \] (6)

\( H_d \) represents the approximate magnitude response of designed filter and \( H_i \) represents the magnitude response of the ideal Band stop(BS) filter.

\[ H_i(\omega_k) = \begin{cases} 0 & \text{for } \omega_0 \leq \omega \leq \omega_{ph} ; \\ 1 & \text{otherwise} \end{cases} \] (7)

\( \omega_{pl} \) and \( \omega_{ph} \) are the cut of frequencies of band stop filter. The absolute error of the magnitude response of FIR filter is given by the equation below

\[ e_1(x) = \sqrt{\sum_{k=0}^{N} \left[H_d(\omega_k, x) - |H_i(\omega_k, x)|\right]^2} \] (8)

The mean squared error of the magnitude response of FIR filter is given by the equation below

\[ e_2(x) = \sqrt{\left[\sum_{k=0}^{N} \left[H_d(\omega_k, x) - |H_i(\omega_k, x)|\right]\right]^2} \] (9)

The desired magnitude response of FIR filter is given as

\[ H_d(x_k) = \begin{cases} 1 & \text{for } \omega_k \in \text{passband} \\ 0 & \text{for } \omega_k \in \text{stopband} \end{cases} \] (10)

The Ripple magnitude of pass band and stop band which are to be minimized are described below

\[ \delta_p = \max |H(\omega_k, x)| - \min |H(\omega_k, x)| \text{ for } \omega_k \in \text{passband} \] (11)
3. OVERVIEW ON DIFFERENTIAL EVOLUTION

Differential Evolution (DE) Algorithm is an evolutionary computational algorithm that was originally introduced by Storn and Price in 1995. The authors developed Differential Evolution algorithm to be a reliable and versatile function optimizer that is readily applicable to a wide range of optimization problems. There are number of significant advantages when using Differential Evolution Algorithm, which are summarized by Price; Ability in many cases to find the true global minimum regardless of the initial parameter values, Fast and simple with regard to application and modification, Requires few control parameters, Parallel processing nature and fast convergence, Capable of providing multiple solutions in a single run, Effective on integer, discrete and mixed parameter optimization, Ability to find the optimal solution for a non-linear constrained optimization problem with penalty functions. Differential Evolution Algorithm is evolutionary approach comprises of four steps: (a) Size of population P; (b) Dimension D (length) of an individual (variables); (c) Mutation factor; (d) Crossover probability CR. In DE, N solution vectors are randomly created at the begin. Mutation, Crossover and Selection operations are applied on population to improve the performance. [6,20]

DE is a direct search method that employs population size P, consisted of floating point encoded individuals (14). During optimization process, at every generation G algorithm maintains a population P^{(G)} of vectors 
\[ x^{(G)} = [x_1^{(G)}, ..., x_i^{(G)}, ..., x_P^{(G)}] \]  
(14)

Each individual \( X_i \) is a D-dimensional vector, containing many parameters (15)
\[ x_i^{(G)} = [x_{i_1}^{(G)}, ..., x_{i_j}^{(G)}, ..., x_{i_D}^{(G)}] \quad i = 1, \ldots, P \]
(15)

3.1. Parameter Setup

To control the DE algorithm the selected key parameters are Population Size (P), boundary constraints of optimization variables (D), Mutation factor (Fm), and Crossover rate (CR), maximum number of iterations G\text{max}.

3.2. Initialisation

In this step the population of individuals is initialised and trial vector is generated. Every vector of initial population is assigned a randomly chosen value within its corresponding feasible bounds.
\[ x_{j_i}^{(G)} = x^\min_{j_i} + \text{rand}(0,1)(x^\max_{j_i} - x^\min_{j_i}) \]
(16)

Where \( i=(1,2,\ldots,P) \) and \( j=(1,2,\ldots,D) \). \( x_{j_i}^{(G)} = 0 \) is the initial value (G=0) of the \( j \)th parameter of the \( i \)th individual vector. The upper and lower bounds of the jth decision parameter are \( x^\max_{j_i} \) and \( x^\min_{j_i} \). Once every vector of the population has been initialised, its corresponding fitness value is calculated and stored for future reference. In this paper Population Size is varied from 50 to 150.

3.3. Mutation

When trial vector is generated. Mutation operation is performed by adding a vector differential to a population vector if individuals. At every generation G, each vector in the population has to serve once as a target vector \( x_j^{(G)} \).

The mutation operator generates mutant vectors \( x_j^{(G)} \) by randomly selected vectors. Five mutation strategies explored are defined below.

**Mutation Strategy 1:**
\[ v_i^{(G+1)} = x_{best}^{(G)} + F(x_{best}^{(G)} - x_i^{(G)}) \]
(17)

**Mutation Strategy 2:**
\[ v_i^{(G+1)} = x_{best}^{(G)} + F(x_{best}^{(G)} - x_i^{(G)} - x_j^{(G)} - x_k^{(G)}) \]
(18)

**Mutation Strategy 3:**
\[ v_i^{(G+1)} = x_{best}^{(G)} + \lambda(x_{best}^{(G)} - x_i^{(G)}) + F(x_i^{(G)} - x_j^{(G)}) \]
(19)

**Mutation Strategy 4:**
\[ v_i^{(G+1)} = x_{best}^{(G)} + F(x_{best}^{(G)} - x_i^{(G)} - x_j^{(G)} - x_k^{(G)}) \]
(20)
Mutation Strategy 5:-

\[ v_i^{(G+1)} = x_i^{(G)} + f \left( x_i^{(G)} - x_j^{(G)} - x_k^{(G)} + x_l^{(G)} \right) \]  \hspace{1cm} (21)

where \( i = 1, 2, \ldots, P \). Vector \( r_1, r_2, r_3 \) and \( r_4 \) are random chosen, where \( r_1 \neq r_2 \neq r_3 \neq r_4 \neq i \). F is user defined constant known as the “scaling mutation factor”. It factor is varied from 0.6 to 1.0 values.

3.4. Crossover

Crossover is applied after mutation process to obtain the trial vector \( U_i \) by mixing the parameters of the mutant vectors \( v_i^{(G+1)} \) and target vector \( x_i^{(G)} \).

\[ U_i^{(G+1)} = \begin{cases} v_i^{(G+1)} & \text{if } \text{rand}[0,1] \leq \text{CR} \text{or } j = s \\ x_i^{(G)} & \text{otherwise} \end{cases} \]  \hspace{1cm} (22)

Where CR is the crossover rate in the range [0, 1]. CR decides whether the random number is copied from mutant vector or trial vector into trial vector. Crossover Rate is selected by varying its values from 0.1 to 0.3.

3.5. Selection

This is the last stage of the DE. The selection operator decides trial vector \( U_i^{(G)} \) should be taken for next generation or the target vector \( x_i^{(G)} \). Thus if \( (f) \) denotes the objective function under minimization then

\[ x_i^{(G+1)} = \begin{cases} U_i^{(G+1)} & \text{if } (U_i^{(G+1)}) \leq (x_i^{(G)}) \\ x_i^{(G)} & \text{otherwise} \end{cases} \]  \hspace{1cm} (23)

In this case both the vectors are compared to each other, the one with minimum objective function is selected for next generation. If the trial vector is better than target vector then trial vector replaces the next generation or else target vector is allowed for next generation.[4, 6, 15, 18]

Algorithm: FIR Filter Design Using Differential Evolution Algorithm

1. Set up all the required control parameters of the DE optimization process.
2. Set iteration \( G=0 \) for initialization.
3. Population \( P \) of individuals are initialized according to equation (16) and
4. Calculate and evaluate the fitness values of the initial individuals according to the problem’s fitness function
5. Rank the initial individuals according to their fitness.
6. \( G=G+1 \) for optimization step of DE algorithm.
7. Apply Mutation operation to generate mutant vector \( v_i^{(G)} \) according to Eq. (17) with a selected mutation strategy.
8. Apply Crossover operation to generate trial vector \( U_i^{(G)} \) according to Eq. (18).
9. Apply Selection operation using Eq. (19) by comparing the target vector and the trial vector. The one with best solution is selected for next generation.
10. Calculate and evaluate the fitness values of individuals according to the problem’s fitness function.
11. Rank new individuals by their fitness.
12. Update the best fitness function value of the current iteration by best fitness value of the previous iteration.
13. Iteration \( G<\text{G}_{\text{max}} \), set \( G=G+1 \) and return to 7 for repeating to search the solution otherwise go to step 14. Stop.

4. SIMULATION RESULT

This section presents the simulation works population size, mutation factor, crossover rate of DE have been optimized for the design of FIR band stop filters. Five different strategies have been explored used on different Parameters used for the design of Band Stop using DE algorithm. The parameter with minimum objective function are selected and used for the design of Band Stop FIR Filter. The Band Stop FIR digital filter is designed by 200 equally spaced points within frequency domain \([0, \pi]\). The design condition for the Band Stop filter design is given in Table 1.

<p>| Table 1: Design Condition for Band Stop FIR Digital Filter |
|---------------------------------|----------------|----------------|----------------|</p>
<table>
<thead>
<tr>
<th>Filter Type</th>
<th>Pass band</th>
<th>Stop band</th>
<th>Max value of H((\omega, x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Band stop</td>
<td>(0 \leq \omega \leq 0.25\pi)</td>
<td>(0.4\pi \leq \omega \leq 0.6\pi)</td>
<td>1</td>
</tr>
</tbody>
</table>

Differential Evolution (DE) algorithm for mutation strategy 1 has been implemented for various filter orders. The order of the filter has been varied from 12 to 32. The algorithm is run 100 times for each order and the results have been recorded accordingly. The main objective of this paper is to minimize the objective function. The graph shows that when the order increases
the objective function decreases. At 26 Filter order objective function is 1.679148 value which is minimum. At order 26 Magnitude Error 1 is 0.955211, Magnitude Error 2 is 0.145541, Pass band value is 1.002822, Stop band value is 0.036107. But at order 28 the objective function increases abruptly. The objective function of order 26 is minimum. So, order 26 is selected for design of band stop FIR filter.

The filter order is taken as 26, which results in number of coefficients as 27. But only half the coefficients are calculated to design because of linear phase property coefficients are symmetrical as shown in Table 2.

**Table 2: Best Optimized Filter Coefficients obtained for Designing Band Stop FIR Filter**

<table>
<thead>
<tr>
<th>h(n)</th>
<th>Filter Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>h(0)=h(26)</td>
<td>0.000407</td>
</tr>
<tr>
<td>h(1)=h(25)</td>
<td>-0.006786</td>
</tr>
<tr>
<td>h(2)=h(24)</td>
<td>-0.000644</td>
</tr>
<tr>
<td>h(3)=h(23)</td>
<td>0.026970</td>
</tr>
<tr>
<td>h(4)=h(22)</td>
<td>0.000674</td>
</tr>
<tr>
<td>h(5)=h(21)</td>
<td>-0.039081</td>
</tr>
<tr>
<td>h(6)=h(20)</td>
<td>-0.005528</td>
</tr>
<tr>
<td>h(7)=h(19)</td>
<td>-0.004074</td>
</tr>
<tr>
<td>h(8)=h(18)</td>
<td>-0.000573</td>
</tr>
<tr>
<td>h(9)=h(17)</td>
<td>0.123823</td>
</tr>
<tr>
<td>h(10)=h(16)</td>
<td>0.000255</td>
</tr>
<tr>
<td>h(11)=h(15)</td>
<td>-0.266419</td>
</tr>
<tr>
<td>h(12)=h(14)</td>
<td>-0.668969</td>
</tr>
<tr>
<td>h(13)</td>
<td>-0.000103</td>
</tr>
</tbody>
</table>

In this paper, five mutation strategies have been applied. As minimum objective function is obtained at Order 26. So, Order 26 is applied on all Mutation strategies given below in Table 3. Objective function with mutation strategy 3 is minimum i.e. is 1.671225 and other parameters are applied.

**Table 3: Objective function for different Strategies**

<table>
<thead>
<tr>
<th>Mutation Strategies</th>
<th>Mutation Strategy 1</th>
<th>Mutation Strategy 2</th>
<th>Mutation Strategy 3</th>
<th>Mutation Strategy 4</th>
<th>Mutation Strategy 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective Function</td>
<td>1.6791 48</td>
<td>2.5697 09</td>
<td>1.6712 25</td>
<td>2.5697 09</td>
<td>2.7131 92</td>
</tr>
</tbody>
</table>

The population size has been varied from 50 to 150 and best objective function is obtained at 140 Population size. So, Population 140 is selected to design Band Stop digital FIR filter and objective function value is 1.648211. Fig.2 shows the graph for Population Size. Mutation factor (fm) has been varied from 0.6 to 1.0. The minimum value of objective function is 1.648211 achieved at 0.8 Mutation factor. The Fig.3 shows the mutation factors values at different objective functions. For the Crossover over selection the values are varied from 0.1 to 0.3. The minimum objective function has been obtained at 0.2 of value 1.648211. So, Crossover rate 0.2 is selected to design Band Stop FIR filter. Figure 4 shows the graph of Crossover Rate and their objective functions.

**Figure 2: Objective function vs Population size**
After obtaining the values for different parameters, the graph of Magnitude Response with normalised frequency has been obtained at Order 26 for Band Stop digital FIR filter. Figure 5 shows the graph for variation in Magnitude Response with variation in Normalised Frequency. Figure 6 shows the graph of Magnitude Response with variation in Normalised Frequency in db for Order 26 to design Band Stop filter.

Table 4: Maximum, Minimum, Average value of objective function with Mutation Strategy 3 along with Standard Deviation.

<table>
<thead>
<tr>
<th>Maximum Value</th>
<th>Minimum Value</th>
<th>Average Value</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.96582</td>
<td>0.93690</td>
<td>0.957383</td>
<td>0.009259</td>
</tr>
</tbody>
</table>

5. CONCLUSION
In this paper, Differential Evolution (DE) algorithm is applied to design Band Stop FIR digital filter. Five different strategies have been used to achieve the objectives of this paper. The simulation results obtained by proposed DE are better in magnitude error and ripple magnitude in pass band and stop band at order 26 with mutation strategy 3 algorithm, population size 140, mutation factor 0.8, crossover rate 0.2. The FIR filter
will perform better and faster due to less number of iterations. So, DE algorithm is powerful and useful design of FIR digital filter. By using DE algorithm we can design various filters such as High Pass, Low Pass and Band Pass. Standard deviation value is 0.009259 which is less than 1. So, the obtained result are robustuous and rugged in nature.

REFERENCES
