SMARANDACHE –BOOLEAN – NEAR –RINGS AND ALGORITHMS WITH EXAMPLES

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ABSTRACT

In this paper we introduced Smarandache-2-algebraic structure of Boolean-near-ring namely Smarandache-Boolean-nearring. A Smarandache-2-algebraic structure on a set N means a weak algebraic structure A0 on N such that there exists a proper subset M of N, which is embedded with a stronger algebraic structure A1, stronger algebraic structure means satisfying more axioms, by proper subset one understands a subset different from the empty set, form the unit element if any, from the whole set. We define Smarandache-Boolean-near-ring and obtain some of its algorithms through Boolean-ring with left-ideals, direct summand, Boolean-l-algebra, Brouwerian algebra and Compatibility. We refer to G. Pilz.

The study of Boolean-near-ring is one of the generalized structure of rings. The study and research on near-rings is very systematic and continuous. Near-rings newsletters containing complete and updated bibliography on the subject of near-rings are published periodically by a team of editors. Then motivated by several researchers we wish to study and analyse the substructure in Smarandache-near-rings. The substructure in near-rings play vital role in the study of near-rings. Unlike other algebraic structure we see in case of near-rings we have the substructure playing vital role in the study and analyse of near-rings. Apart from the sub near-rings and ideals of near-rings we have special substructure like N-groups, filter and modularity in near-rings. It is this study in the context of Smarandache-Boolean-near-rings will yield several interesting results. Also the Smarandache substructure in Boolean-near-rings will also yield very many results in the direction.

For the study we would be using the book of Pilz Gunter, Near-rings (1997) published by North Holland Press, Amsterdam[10], Special Algebraic Structure by FlorentinSmarandache, University of New Mexico, USA (1991)[18 ], Smarandache Algebraic Structure by Raul Padilla, Universidade do Minho, Portugal (1999) [13], Blackett [3] discusses the near-ring of affine transformations on a vector space where the near-ring has a unique maximal ideal. Gonshor [8] defines abstract of affine near-rings and completely determines the lattice of ideals for these near-rings. The near-rings of differential transformations is seen in [4]. For near-rings with geometric interpretation [10] or [18] and several research papers on Boolean-near-rings. We would first study and characterize the ideals and sub Boolean-near-rings in Smarandache-Boolean-near-rings. Also to study and analyse those Boolean-near-rings, which are Smarandache-Boolean-near-ring and find the conditions for Smarandache-Boolean-near-rings. Yet another major substructure in Boolean-near-rings is the notion of filters. We would extend and study the notion of Smarandache filters given in Smarandache-Boolean-near-rings.

Further the notion of Smarandache ideals in near-ring would be studied, characterized and analysed for Smarandache-Boolean-near-rings. Both the notions viz. N-groups and ideals in near-ring and Smarandache-Boolean-near-rings would be compared and contrasted. Also the nice notion of modularity in near-rings, which are basically built using concepts of idempotents, will be studied and analysed in Smarandache modularity in Boolean-near-ring. Finally, Smarandache-Boolean-near-rings has constructed from Boolean-ring by an algorithmic approach through its substructures and Smarandache-Boolean-near-ring has introduced some application.

Keywords: Boolean-ring, Boolean-near-ring, S-Boolean-near-ring, Boolean-l-algebra, Brouwerian algebra and Compatibility.

1. INTRODUCTION

In order that New notions are introduced in algebra to better study the congruence in number theory by FlorentinSmarandache [4]. By \(<\)proper subset\> of a set A we consider a set P included in A, and different from A, different form the empty set, and from the unit element in A – if any they rank the algebraic structures using an order relationship:

They say that the algebraic structures S1<< S2 if: both are defined on the same set; all S1 laws are also S2 laws; all axioms of an S1 law are accomplished by the corresponding S2 law; S2 law accomplish strictly more axioms that S1 laws, or S2 has more laws than S1.

For example: Semi group <<Monoid<< group << ring<< field, or Semi group<< commutative semi group, ring<< unitary, ring etc. They define a General special structure to be a structure SM on a set A, different form a structure SN, such that a proper subset of A is a structure, where SM<< SN <<.

2. PRELIMINARIES

DEFINITION: 2.1

A left near-ring A is a system with two binary operations, addition and multiplication, such that

(i) the elements of A form a group (A,+) under addition,
(ii) the elements of A form a multiplicative semi-group,
(iii) x(y + z) = xy + xz, for all x,y,z \in A

In particular, if A contains a multiplicative semi-group S whose elements generate (A,+) and satisfy

(iv) (x + y) s = xs + ys, for all x, y \in A and s \in S, then we say that A is a distributively generated near-ring.

DEFINITION: 2.2

A near-ring (B, +, •) is Boolean-Near-Ring if there exists a Boolean-ring (A,+, •, 1) with identity such that • is defined in terms of +, \wedge and 1, and for any b \in B, b • b = b

DEFINITION: 2.3

A near-ring (B, +, •) is said to be idempotent if x^2 = x, for all x \in B. If (B,+, •) is an idempotent ring, then for all a, b \in B, a + a = 0 and a • b = b • a

DEFINITION: 2.4

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A Boolean-near-ring \((B, +, \circledast)\) is said to be Smarandache-Boolean-near-ring whose proper subset \(A\) is a Boolean-ring with respect to same induced operation of \(B\).

**DEFINITION: 2.5 (Alternative definition for S-Boolean-near-ring)**

If there exists a non-empty set \(A\) which is a Boolean-ring such that it superset \(B\) of \(A\) is a Boolean-near-ring with respect to the same induced operation, then \(B\) is called Smarandache-Boolean-near-ring.

It can also written as S-Boolean-near-ring.

**EXAMPLE FOR SMARANDACHE-BOOLEAN-NEAR-RING: 2.6**

**Boolean-ring:**

A Boolean-ring is an algebraic structure \((A, +, \circledast)\) together with two binary operations addition and multiplication defined as follows

\((A, +)\) is a group,

For, (i) Closure under addition :

For all \(a, b \in A\), then \(a + b \in A\)

(ii) Associativity under addition :

For all \(a, b, c \in A\), then \((a + b) + c = a + (b + c) \in A\)

(iii) Commutativity of addition :

For all \(a, b \in A\), then \(a + b = b + a \in A\)

(iv) Identity element for addition :

For all \(a \in A\), then there exists \(0 \in A\) such that \(0 + a = a + 0 = a \in A\)

(v) Characteristic 2 for addition :

For all \(a \in A\), then \(a + a = 0 \in A\)

\((A, \circledast)\) is a semigroup :

(vi) Closure under product :

For all \(a, b \in A\), then \(a \circledast b \in A\)

(vii) Associativity of product :

For all \(a, b, c \in A\), then \((a \circledast b) \circledast c = a \circledast (b \circledast c) \in A\)

(viii) Identity element for product :

For all \(a \in R\) then there exists \(1 \in R\) such that \(1 \circledast a = a \circledast 1 = a \in A\)

(ix) Idempotent of product : For all \(a \in A\), then \(a \circledast a = a \in A\)

Product is distributive over addition : 

(x) Left-distributive law holds: For all \(a, b, c \in A\), then 

\[a \circledast (b + c) = (a \circledast b) + (a \circledast c) \in A\]

**Example for Boolean-ring :**

Let \(A = \{0, a\} \subseteq B\), be a finite-Boolean-ring. Defined by

\[
\begin{array}{ccc}
+ & 0 & a \\
0 & 0 & a \\
a & a & 0 \\
\end{array}
\]

**Boolean-near-ring :**

A near-ring \((B, +, \circledast)\) is said to be Boolean-near-ring if there exists a Boolean-ring \((A, +, \wedge, 1)\) with that that \(\circledast\) terms \(+, \wedge\) and \(1\), and for any \(b \in B\),

\[b \circledast b = b\]

\[
\begin{array}{ccc}
+ & 0 & a & b & c \\
0 & 0 & a & b & c \\
a & a & 0 & b & c \\
b & b & c & 0 & a \\
c & c & b & a & 0 \\
\end{array}
\]

\[
\begin{array}{ccc}
\circledast & 0 & a & b & c \\
0 & 0 & a & b & c \\
a & a & a & a & a \\
b & b & b & b & b \\
c & c & c & c & c \\
\end{array}
\]

Therefore, the above conditions are satisfied, then we write \(B\) is a Boolean-near-ring.

**Smarandache-Boolean-near-ring :**

A Boolean-near-ring \((B, +, \circledast)\) is said to be Smarandache-Boolean-near-ring whose proper subset \(A\) is a Boolean-ring with respect to the same induced operation of \(B\).

Verify that, \(B\) is a Smarandache-Boolean-near-ring under ‘+’ and ‘\(\circledast\)’, 

For check the following conditions,

\((B, +)\) is a group,

For,  (i) Closure under addition :
For all $a, b \in B$, then $a + b \in B$

(ii) Associativity under addition:

For all $a, b, c \in B$, then $(a + b) + c = a + (b + c) \in B$

(iii) Commutativity of addition:

For all $a, b \in B$, then $a + b = b + a \in B$

(iv) Identity element for addition:

For all $a \in B$, then there exists $0$ in $B$ such that $0 + a = a + 0 = a \in B$

(v) Characteristic $2$ for addition:

For all $a \in B$, then $a + a = 0 \in B$

(B, $\cdot$) is a semigroup:

(vi) Closure under product:

For all $a, b \in B$, then $a \cdot b \in B$

(vii) Associativity of product:

For all $a, b, c \in B$, then $(a \cdot b) \cdot c = a \cdot (b \cdot c) \in B$

(viii) Identity element for product:

For all $a$ in $R$ then there exists $1$ in $R$ such that $1 \cdot a = a \cdot 1 = a \in B$

(ix) Idempotent of product:

For all $a \in A$, then $a \cdot a = a \in B$

Product is distributive over addition:

(x) Left-distributive law holds, for all $a, b, c \in B$, then $a \cdot (b + c) = (a \cdot b) + (a \cdot c) \in B$

Here $A$ is satisfied idempotent condition of $B$, then we write $A$ is a Boolean-ring, for $a$ in $A$ then $a \cdot a = a \in A$

Hence, the proper subset of Boolean-near-ring is a Boolean-ring and therefore, $B$ is a Smarandache-Boolean-near-ring.

3. **ALGORITHMIC STRUCTURE OF SMARANDACHE-BOOLEAN-NEAR-RING:**

In this section, Algorithms to construct the S-Boolean-near-rings from its characterization are obtained.

**THEOREM : 3.1**

If a non-empty set $B$ contains a unique minimal Boolean-$l$-algebra contained in all other non-zero Boolean-$l$-algebras. Then $B$ is a Smarandache-Boolean-near-ring.

**proof:** Consider a Boolean-ring $I_0 \neq \{0\}$, since a Boolean-ring itself is a Boolean-$l$-algebra. Then $I_0$ is a Boolean-$l$-algebra. Let $B$ be non-empty set in which $A$ is a proper subset. Now to find subsets of $B$ which contains $I_0$ such that they are Boolean-$l$-algebras with respect to the same induced operations of $A$. In Gunter Pilz [4] in section 1.60. The Theorem by Gratzer and Fain is given by the following conditions for a Boolean-near-ring $B \neq \{0\}$ are equivalent

1. $\bigcap I \neq \{0\}, \{0\} \neq I \subseteq N$

2. $B$ contains a unique minimal Boolean-$l$-algebra, contained in all other non-zero Boolean-$l$-algebras.

Hence, consider the non-empty set $B$ is a Boolean-near-ring. Now by Theorem [1], $B$ is a Smarandache-Boolean-near-ring.

By similar argument by the Theorem[1], We have the following results

**THEOREM : 3.2**

If a non-empty set $B$ contains a unique minimal Browerian algebra contained in all other non-zero Browerian algebras. Then $B$ is a Smarandache-Boolean-near-ring.


**THEOREM : 3.3**

If a non-empty set $B$ contains a unique minimal compatibility contained in all other non-zero compatibilities. Then $B$ is a Smarandache-Boolean-near-ring. Also by theorem [1], we have the following results

**ALGORITHM : 3.1.1**

**BOOLEAN-$l$-ALGEBRA**

**Step 1:** Consider a Boolean-ring $A$

**Step 2:** Verify that $A$ is a Boolean-ring with respect to same induced operations

For, Check the following conditions,

‘$+$’ is defined as follows,

1. For all $n_1, n_2 \in A$, then $n_1 + n_2 \in A$

2. For all $n_1, n_2, n_3 \in A$, then $n_1 + (n_2 + n_3) = (n_1 + n_2) + n_3$

3. For all $n \in A$, there exist $e \in A \Rightarrow n + e = e + n = n$

4. For all $n \in A$, there exist $n' \in A \Rightarrow n + n' = n' + n = e$

Let $A^* = A \setminus \{0\}$

5. For all $n_1 \in A^* \Rightarrow n_1 \cdot n_1 = n_1 \in A^*$,

6. For all $n_1, n_2, n_3 \in A^*$, then $n_1 \cdot (n_2 \cdot n_3) = (n_1 \cdot n_2) \cdot n_3$

7. For all $n \in A^*$, there exist $e \in A \Rightarrow n \cdot e' = e' \cdot n = n$

8. For all $n \in A^*$, there exist $n' \in A \Rightarrow n \cdot n' = n' \cdot n = e'$
9. For all $n \in A^*$, then $n \cdot n = n$
10. For all $n, m \in A^*$, then $n \cdot m = m \cdot n$
11. For all $n_1, n_2, n_3 \in A^*$, then $n_1 + (n_2 \cdot n_3) = (n_1 + n_2) \cdot n_3$
12. For all $n_j \in A \Rightarrow n_1 + n_j = 0$

The above conditions are satisfied, then we write $(B, +, \cdot)$ is a Boolean-ring.

Step 3: Let $I_i, i = 0, 1, 2, 3$ be supersets of $I_0$.

Step 4: Let $B = \bigcup I_i$

Step 5: Choose the sets $I_j$ from $I_i$’s subject to $i_j \subseteq i_j$ implies $i_j \bigcap (i_j \bigcap i_j) = 0$

Step 6: Verify that $\bigcap I_j = I_0 \neq \{0\}$

Step 7: If step (6) is true, then write $B$ is a Smarandache-Boolean near-ring.

EXAMPLE: BOOLEAN-ALGEBRA 3.1.2

Step 1: Consider a non-empty set $A = \{0, n_1\}$

Step 2: Verify that $A = \{0, n_1\}$ is a Boolean-ring with respect to same induced operations

For, Checking the following conditions,

- ‘+’ is defined as follows,
  $0 + 0 = 0, \ 0 + n_1 = n_1, \ n_1 + 0 = n_1, \ n_1 + n_1 = 0$
  (i) Closure law: For all $n_1 \in A$, $0 + 0 = 0 \in A$, $0 + n_1 = n_1 \in A, \ n_1 + 0 = n_1 \in A$
  $n_1 + n_1 = 0 \in A$

- ‘?’ is defined as follows,
  $0 + (0+n_1) = (0+0) + n_1 = n_1 \cdot n_1 = n_1$
  $0 + n_1 = n_1 + 0 \Rightarrow n_1 + n_1 = n_1 + n_1$

- ‘?’ is additive identity element: For all $n_1 \in A$

  $0 + n_1 = n_1 + 0 = n_1, \ 0 + 0 = 0 = n_1$

- ‘?’ is additive identity element: For all $n_1 \in A$

  $0 = 0 = 0$

- ‘?’ is defined by $0 = 0 = 0 = 0 = 0$

- ‘?’ is defined by $0 = 0 = 0 = 0 = 0$

- ‘?’ is defined by $0 = 0 = 0 = 0 = 0$

Therefore, the commutativity satisfied under addition.

(vi) Characteristic 2 for addition is defined as,

- For all $0, n_1 \in A, \ 0 + 0 = 0, \ n_1 + n_1 = 0$

Now ‘?’ is defined as follows:

- If $n_1 \leq n_1$ then $n_1 \cap n_1 = n_1$ and $n_1 \cap n_1 = n_1$

(vii) Closure law: For all $n_1 \in A, \ 0 \cap 0 = 0, \ 0 \cap n_1 = n_1$

(viii) Associative law: For all $n_1 \in A, \ 0 \cap (0 \cap 0) = (0 \cap 0) \cap 0$

(xi) Commutative law: For all $n_1 \in A, \ 0 \cap 0 = 0, \ n_1 \cap 0 = 0 = 0$,

(xii) Distributive law holds.

Therefore, $(A, +, \cap)$ is a Boolean-ring.

Step 3: Let $A = A_0 = I_0$

Let $I_0 = A = \{0, n_1\}$

Step 4: Consider the supersets $I_i, \ i = 0, 1, 2, 3$ of $I_0$.

$I_0 = \{0, n_1\}$

$I_1 = \{0, n_1, n_2\}$

$I_2 = \{0, n_1, n_3\}$

$I_3 = \{0, n_1, n_2, n_3\}$

Step 5: Let $B = \bigcup_{I_i \subseteq I_j}$

Step 6: Choose set $I_j$’s from $I_i$’s subject to $n_1 \leq n_j$, implies $n_1 \cap (n_j - n_1) = 0$, for all $n_1, n_j \in I_j$

Step 7: $(B, +, \cdot, \cap, \cup)$ is defined as follows:

- ‘+’ is defined by $0 + 0 = 0, \ 0 + n_1 = n_1, \ 0 + n_2 = n_2, \ 0 + n_3 = n_3$

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a-b is defined by a-b = a+(a \bigcap b)
For all 0 \in B, \ n_i \in I_0 \ then
0 \bigcap (n_1 - 0) = 0 \bigcap (n_1 + n_2 \bigcap 0) / \n= 0 \bigcap (n_1 + 0)
= 0 \bigcap n_1
= 0 \in I_0

For all \ n_i \in B, \ 0 \in I_0 \ then
n_1 \bigcap (0 - n_1) = n_1 \bigcap (0 + 0 \bigcap n_1)
= n_1 \bigcap (0 + 0)
= n_1 \bigcap 0
= 0 \in I_0

For all \ n_1, 0 \in B, \ then,
n_2 \bigcap (0 - n_1) = n_2 \bigcap (0 + 0 \bigcap n_1)
= n_2 \bigcap (0 + 0)
= n_2 \bigcap 0
= 0 \in I_0

For all \ n_i \in B, \ 0 \in I_0 \ then,
n_3 \bigcap (0 - n_1) = n_3 \bigcap (0 + 0 \bigcap n_1) = n_3 \bigcap (0 + 0)
= n_3 \bigcap 0
= 0 \in I_0

For all \ n_1 \in B, \ n_i \in I_0 \ then,
n_2 \bigcap (n_1 - n_2) = n_2 \bigcap (n_1 + n_2 \bigcap n_1)
= n_2 \bigcap 0
= 0 \in I_0

For all \ n_1 \in B, \ n_i \in I_0 \ then,
n_3 \bigcap (n_1 - n_3) = n_3 \bigcap (n_1 + n_1 \bigcap n_1)
= n_3 \bigcap (n_1 + n_1)
= n_3 \bigcap 0
= 0 \in I_0

For all \ 0 \in B, \ 0 \in I_0 \ then,
0 \bigcap (0 - 0) = 0 \bigcap (0 + 0 \bigcap 0)
= 0 \bigcap 0
= 0 \in I_0

Hence, \ I_0 \ satisfies all the requirements.
Therefore, we choose \ I_0 \ as \ I_1.

Hence for \ I_1 = \{0, n_1, n_2\} \ and \ B = \{0, n_1, n_2, n_3\}
\ n_1 \bigcap (n_1 - n_1) = n_1 \bigcap (n_1 + n_1 \bigcap n_1)

Similarly, for \ I_2 = \{0, n_1, n_3\} \ and \ B = \{0, n_1, n_2, n_3\}
\ n_1 \bigcap (n_1 - n_3) = n_1 \bigcap (n_1 + n_1 \bigcap n_1)

Similarly, for \ I_3 = \{0, n_2, n_3\} \ and \ B = \{0, n_1, n_2, n_3\}
\ n_1 \bigcap (n_3 - n_2) = n_1 \bigcap (n_3 + n_3 \bigcap n_2)

Step 9: If step (8) is true, then we write \ B = \text{a Smarandache-Boolean-ring}.

ALGORITHM: 3.2.1
BROWERIAN ALGEBRA

Step 1: Consider a Boolean-ring \ A

Step 2: Verify that \ A \ is a Boolean-ring with respect to same induced operations

For, \ Check the following conditions,

‘\ast’ is defined as follows,

1. For all \ n_1, n_2 \in A, \ then \ n_1 \ast n_2 \in A
2. For all \ n_1, n_2, n_3 \in A, \ then \ n_1 \ast (n_2 \ast n_3) = (n_1 \ast n_2) \ast n_3
3. For all \ n \in A, \ there exist \ e \in A \Rightarrow \ n \ast e = e = e \ast n
4. For all \ n \in A, \ there exist \ n' \in A \Rightarrow n \ast n' = n' \ast n = e

Let \ A' = A / \{0\}

5. For all \ n_1 \in A', \ there exist \ n_1 = n_1 \in A', \ n_2 \in A, \ then \ n_1 \ast n_2 \in A'
6. For all \ n_1, n_2, n_3 \in A', \ then \ n_1 \ast (n_2 \ast n_3) = (n_1 \ast n_2) \ast n_3
7. For all \ n \in A*, \ there exist \ e \in A \Rightarrow n \ast e' = e' \ast n = n
8. For all \ n \in A*, \ there exist \ n' \in A \Rightarrow n \ast n' = n' \ast n = e'
9. For all \ n \in A', \ then \ n \ast n = n
10. For all \ n, m \in A', \ then \ n \ast m = m \ast n
11. For all \ n_1, n_2, n_3 \in A', \ then \ n_1 \ast (n_2 \ast n_3) = (n_1 \ast n_2) \ast n_3
12. For all \ n_1 \in A \Rightarrow n_1 + n_1 = 0

The above conditions are satisfied, then write \ (B, \ast, \bullet) \ is a Boolean-ring.

Step 3: Let \ I_1, i = 0, 1, 2, 3, \ldots, \ be the supersets of \ I_0

Step 4: Let \ B = \bigcup \ I_i

Step 5: Choose the sets \ I_i \ from \ I_1 \ \text{subject to for all} \ x \in B \ \text{such that} \ x \leq a \ \text{then} \ a = x \cup (a-x), \ \text{for all} \ x \ \text{and} \ a \in I_i

Step 6: Verify that \ \bigcap I_j = I_0 = \{0\}

Step 7: If step (6) is true, then we write \ B \ is a Smarandache-Boolean-near-ring.

EXAMPLE: BROWERIAN-ALGEBRA 3.2.2

Step 1: Consider a non-empty set \ A = \{0, n_1\}

Step 2: Verify that \ A = \{0, n_1\} \ is a Boolean-ring with respect to same induced operations,

For, \ Check the following conditions,

‘\ast’ is defined as follows,

0 \ast 0 = 0, \ 0 \ast n_1 = n_1, \ n_1 \ast 0 = n_1, \ n_1 \ast n_1 = 0

(i) Closure law : For all \ 0, n_1 \in A,
(ii) Associative law : For all 0, \( n_1 \in A \),
\[
0 + (0 + 0) = (0 + 0) + 0 / \quad n_1 + (n_1 + n_2) = (n_1 + n_1) + n_2
\]

0 + 0 = 0 + 0 / \quad n_1 + n_2 = n_1 + n_2
0 = 0 / \quad n_1 = n_1

0 + (n_1 + 0) = (0 + n_1) + 0 / \quad n_1 + (n_1 + 0) = (n_1 + n_1) + 0
0 + n_1 = n_1 + n_1 / \quad n_1 + 0 = n_1 + 0
0 = 0 / \quad n_1 = n_1

0 + (n_1 + n_2) = (0 + n_1) + n_2 / \quad n_1 + (n_1 + n_2) = (n_1 + n_1) + n_2
0 + 0 = n_1 + n_2 / \quad n_1 + 0 = n_1 + n_2
0 = 0 / \quad n_1 = n_1

(iii) '0' is the additive identity element :
For all 0, \( n_1 \in A \),
\[
0 + n_1 = n_1 + 0 = n_1 / \quad n_1 + 0 = n_1 + n_1
\]

(iv) The inverses of 0, \( n_1 \) are respectively 0, \( n_1 \) in A under addition.
(v) Commutative law : For all 0, \( n_1 \in A \),
\[
0 + 0 = 0 + 0 / \quad 0 + n_1 = n_1 + 0
0 = 0 / \quad n_1 = n_1
n_1 + 0 = n_1 + n_1 / \quad n_1 + 0 = n_1 + n_1
0 = 0 / \quad n_1 = n_1

Therefore, the commutative law satisfied under addition.
(vi) Characteristic 2 for addition is defined as,
For all 0, \( n_1 \in A \),
\[
0 + 0 = 0 / \quad n_1 + n_1 = 0
\]

Now ' \bigcap ' is defined as follows :
If \( n_1 \leq n_2 \) then \( n_1 \cap n_2 = n_1 \) and \( n_1 \cap n_2 = n_2 \),
\[
0 \cap 0 = 0, \quad 0 \cap n_1 = 0, \quad n_1 \cap 0 = n_1, \quad n_1 \cap n_1 = n_1
\]

(vii) Closure law : For all 0, \( n_1 \in A \),
\[
0 \cap 0 = 0, \quad n_1 \cap n_2 = n_1 \cap n_2
\]

(viii) Associative law : For all 0, \( n_1 \in A \),
\[
0 \cap (0 \cap 0) = (0 \cap 0) \cap 0 / \quad 0 \cap 0 = 0 \cap 0
0 = 0 / \quad n_1 = n_1
n_1 \cap (n_1 \cap n_2) = (n_1 \cap n_1) \cap n_2 / \quad n_1 \cap n_1 = n_1 \cap n_1
n_1 = n_1 / \quad n_1 = n_1

Similar way, we proceed another associate laws.
(ix) Commutative law : For all 0, \( n_1 \in A \),
\[
0 \cap 0 = 0 / \quad n_1 \cap n_1 = n_1 \cap n_1
0 = 0 / \quad n_1 = n_1
n_1 \cap 0 = 0 \cap n_1 = 0 \cap n_1 = 0
0 = 0 / \quad n_1 = n_1
n_1 \cap 0 = 0 \cap n_1 = 0 \cap n_1 = 0
0 = 0 / \quad n_1 = n_1
n_1 \cap 0 = 0 \cap n_1 = 0 \cap n_1 = 0
0 = 0 / \quad n_1 = n_1

(x) Idempotent law : For all 0, \( n_1 \in A \),
\[
0 \cap 0 = 0, \quad n_1 \cap n_1 = n_1
\]

(xi) For all 0, \( n_1 \in A \),
\[
0 \cap (0 \cap 0) = (0 \cap 0) \cap (0 \cap 0)
\]
\[
n_1 \cap (n_1 \cap n_1) = (n_1 \cap n_1) \cap (n_1 \cap n_1)
\]

Similar way, the right distributive law holds.

(xii) Define, the complement: For all 0, \( n_1 \in A \),
\[
0 \bigcap n_1 = n_1 \bigcap n_1 = n_1 \bigcap n_1
\]

Therefore, \( (A, +, \cap, \cup) \) is a Boolean-ring.

Step 3 : Let \( A = A_0 = I_0 \)
Let \( I_0 = \{ 0, n_1 \} \)

Step 4 : Consider the supersets \( I_0, i = 0, 1, 2, 3 \) of \( I_0 \).
\[
I_0 = \{ 0, n_1 \}
I_1 = \{ 0, n_1, n_2 \}
I_2 = \{ 0, n_1, n_3 \}
I_3 = \{ 0, n_1, n_2, n_3 \}
\]

Step 5 : Let \( B = \bigcup_{I_3 \neq I_0} I_3 \)

Step 6 : Choose set \( I_0 \)'s from \( I_0 \)'s subject to \( x \leq a \) implies \( a = x \cup (a \setminus x) \), for all \( a, x \in I_0 \)

Step 7 : \( (B, +, \cdot, \bigcup, \bigcap) \) is defined as follows :
\( + \) and \( \bigcup \) is defined as follows,
\( \cdot \) is defined by
\[
0 \cdot 0 = 0, \quad 0 \cdot n_1 = n_1, \quad 0 \cdot n_2 = n_2, \quad 0 \cdot n_3 = n_3
\]
\[ = n_3 \cup (0 + 0) = n_3 \cup 0 = n_3 \quad \forall n \in I_0 \]

For all \( n_2 \in B, n_1 \in I_0 \) then,
\[ n_2 \cup (n_1 - n_2) = n_2 \cup (n_1 + n_3) \]
\[ = n_2 \cup 0 = n_2 \in I_1 \]

For all \( n_1 \in B, n_1 \in I_0 \) then,
\[ n_1 \cup (n_1 - n_1) = n_1 \cup (n_1 + n_1) \]
\[ = n_1 \cup 0 = n_1 \quad \forall n \in I_0 \]

For all \( 0 \in B, 0 \in I_0 \) then,
\[ 0 \cup (0 - 0) = 0 \cup (0 + 0) \]
\[ = 0 \in I_0 \]

Hence, \( I_0 \) satisfies all the requirements.

Therefore, we choose \( I_0 \) as \( I_1 \). Hence for \( I_1 = \{0,n_1,n_2\} \) and \( B = \{0,n_1,n_2, n_3\} \)
\[ n \cup (m - n) = m \quad \forall n, m \in B, \text{ and } 0 \in I_1 \].
Therefore, \( I_1 \) also choose as \( I_1 \).

Similarly, for \( I_2 = \{0,n_1,n_3\} \) and \( B = \{0,n_1,n_2, n_3\} \)
\[ n \cup (m - n) = m \quad \forall n, m \in B, \text{ and } 0 \in I_2 \].
Therefore, \( I_2 \) also choose as \( I_2 \).

Similarly, for \( I_3 = \{0,n_2,n_3\} \) and \( B = \{0,n_1,n_2, n_3\} \)
\[ n \cup (m - n) = m \quad \forall n, m \in B, \text{ and } 0 \in I_3 \].
Therefore, \( I_3 \) becomes \( I_3 \).

Step 8 : Verify that \( \cap I_j = 0 \not\subseteq \{0\} \subseteq B \)
\[ I_0 \cap I_1 \cap I_2 \cap I_3 = \{0, n_1, n_2, n_3\} \cap \{0, n_1, n_2, n_3\} \]
\[ = \{0, n_1\} \not\subseteq \{0\} \subseteq B \]

Step 9 : If step (8) is true, then we write \( B \) is a Smarandache-Boolean-ring.

ALGORITHM : 3.3.1

COMPATIBILITY :
Step 1 : Consider a Boolean-ring \( A \)

Step 2 : Verify that \( A \) is a Boolean-ring with respect to same induced operations
For, Check the following conditions, ‘+’ is defined as follows,
1. For all \( n_1, n_2 \in A \) then \( n_1 + n_2 \in A \)
2. For all \( n_1, n_2, n_3 \in A \), then \( n_1 + (n_2 + n_3) = (n_1 + n_2) + n_3 \)
3. For all \( n \in A \), there exist \( e \in A \) such that \( n + e = n \)
4. For all \( n \in A \), there exist \( n' \in A \) such that \( n + n' = 0 \)

Let \( A' = A / \{0\} \)
5. For all \( n_1 \in A' \Rightarrow n_1 \cdot n_1 = n_1 \in A' \)
6. For all \( n_1, n_2, n_3 \in A' \), then \( n_1 \cdot (n_2 \cdot n_3) = (n_1 \cdot n_2) \cdot n_3 \)
7. For all \( n \in A' \), there exist \( e \in A \) such that \( n + e = n \)
8. For all \( n \in A' \), there exist \( n' \in A \) such that \( n + n' = 0 \)

The above conditions are satisfied, then write \( (B, +, \cdot) \) is a Boolean-ring.

Step 3 : Let \( I_i, i = 0,1,2,3,\ldots \) be the supersets of \( I_0 \)

Step 4 : Let \( B = \bigcup I_i \)

Step 5 : Choose the sets \( I_j \) from \( I_i \)’s subject to for all \( a, b \in I_j \) such that \( ab = ba \in I_j \)

Step 6 : Verify that \( \cap I_j = I_0 \not\subseteq \{0\} \)

Step 7 : If step (6) is true, then we write \( B \) is a Smarandache-Boolean-ring.

EXAMPLE : COMPATIBILITY 3.3.2
Step 1 : Consider a non-empty set \( A = \{0, n_1\} \)

Step 2 : Verify that \( A = \{0, n_1\} \) is a Boolean-ring with respect to same induced operations
For, Check the following conditions,
‘+’ is defined as follows,
\[ 0 + 0 = 0 \]
\[ 0 + n_1 = n_1 + 0 = n_1 \]
\[ n_1 + n_1 = n_1 \]
\[ n_1 + n_1 = n_1 \]

(i)Closure law : For all \( n, n_1 \in A \),
\[ 0 + 0 = 0 \in A \]
\[ 0 + n_1 = n_1 + 0 = n_1 \]
\[ n_1 + n_1 = n_1 \]

(i)Associative law : For all \( n, n_1 \in A \),
\[ (0 + 0) = 0 \]
\[ (n_1 + 0) = n_1 + 0 = n_1 \]
\[ n_1 + n_1 = n_1 + n_1 = n_1 \]

0 + 0 = 0

0 + n_1 = n_1 + 0 = n_1

n_1 = n_1

n_1 = n_1

0 + n_1 = n_1 + n_1 = n_1 + n_1

0 + 0 = n_1 + n_1 = n_1 + n_1

0 + 0 = n_1 + n_1 = n_1 + n_1

0 + 0 = n_1 + n_1 = n_1 + n_1

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0   = 0 / \ n_1 = n_1

(iii) ‘0’ is the additive identity element: For all 0, n_1 \in A
0+n_1 = n_1 + 0 = n_1 / n_1 + 0 = 0 + n_1 = n_1

(iv) The inverses of 0, n_1 are respectively 0, n_1 in A under addition.

(v) Commutative law: For all 0, n_1 \in A
0 + n_1 = n_1 + 0 / n_1 = n_1
n_1 + n_2 = n_2 + n_1 / n_1 + n_2 = n_2 + n_1

Therefore, the commutativity satisfied under addition.

(vi) Characteristic 2 for addition is defined as,
For all 0, n_1 \in A,
0 + 0 = 0 / n_1 + n_1 = 0

Let A’ = A / \{0\} = \{0, n_1\} = n_1,
Now ‘•’ is defined as follows,
0 • 0 = 0, 0 • n_1 = 0, n_1 • 0 = 0, n_1 • n_1 = n_1

In general, we define n_1 • m = n_1, for all n, m \in A

(vii) For all n_1 \in A’, n_1 + n_1 = n_1 \in A’,
(viii) For all n_1 \in A’,
A’ = n_1 • n_1 = (n_1 • n_1) • n_1
n_1 • n_1 = n_1 \quad / 
\quad n_1 = n_1

(ix) ‘n_1’ is the identity element of A’ under multiplication

(x) For all 0, n_1 \in A’,
0 • 0 = 0 / n_1 • n_1 = n_1

Hence, the idempotent law is satisfied under multiplication.

(xi) Product is distributive over addition as follows:
For all 0, n_1 \in A,
0 \cdot (0+0) = 0 \cdot 0 + 0 \cdot 0 / \quad 0 \cdot 0 = 0 + 0
0 \cdot n_1 = 0 + n_1
0 = 0
n_1 • (n_1 + n_1) = (n_1 • n_1) + (n_1 • n_1)
\quad n_1 • 0 = n_1 + n_1
0 = 0
\quad n_1 • (0+0) = n_1 • 0 + n_1 • 0
n_1 • 0 = 0 + n_1
\quad n_1 = 0
\quad 0 = 0
n_1 • (n_1 + 0) = n_1 • n_1 + n_1 • 0 / 0 • (n_1 + 0) = 0 • n_1 + 0 • n_1
n_1 • n_1 = n_1 + 0 / 0 • 0 = 0 + 0
n_1 = 0
n_1 • (0 + 0) = 0 • n_1 + 0 • 0 / 0 • (0 + n_1) = 0 • 0 + 0 • n_1
0 • n_1 = 0 + 0 / 0 • n_1 = 0 + 0
0 = 0 / 0 = 0

Similar way, the right distributive law holds.

(xii) Commutative law: For all 0, n_1 \in A
0 • 0 = 0 \cdot 0 / 0 • n_1 = n_1 • 0
0 = 0 / 0 = 0

Hence, all the requirements of Boolean-rings are satisfied.
Therefore, (A, +, •) is a Boolean-ring.

Step 3: Let A = A_0 = I_0
Let I_0 = \{0, n_1\}

Step 4: Consider the super sets I_i, i \in \{0, 1, 2, 3\} of I_0.
I_0 = \{0, n_1\}
I_1 = \{0, n_1, n_2\}
I_2 = \{0, n_1, n_2\}
I_3 = \{0, n_1, n_2, n_3\}

Step 5: Let B = \bigcup_{i=0}^{3} I_i

Step 6: Choose set I_i’s from I_i’s subject to for all a, b \in I_j such that \ ab^2 = a^2b \in I_j

Step 7: (B, +, •) is defined as follows:
• + and • is defined as follows,
‘+’ is defined by
\quad 0+0 = 0, 0+n_1 = n_1, 0+n_2 = n_2, 0+n_3 = n_3,
n_1+n_1 = n_1, n_1+n_2 = n_2, n_1+n_3 = n_3,
n_2+n_1 = n_2, n_2+n_2 = n_3, n_2+n_3 = n_4,
n_3+n_1 = n_3, n_3+n_2 = n_3, n_3+n_3 = n_3,

‘•’ is defined by
\quad 0 • 0 = 0, 0 • n_1 = 0, 0 • n_2 = 0, 0 • n_3 = 0,
n_1 • 0 = n_1, n_1 • n_1 = n_1, n_1 • n_2 = n_1, n_1 • n_3 = n_1,
n_2 • 0 = n_2, n_1 • n_1 = n_2, n_1 • n_2 = n_2, n_1 • n_3 = n_2,
n_3 • 0 = n_3, n_1 • n_2 = n_3, n_2 • n_1 = n_3, n_2 • n_3 = n_3,
n_3 • 0 = n_3, n_3 • n_1 = n_3, n_3 • n_2 = n_3, n_3 • n_3 = n_3,

nm^2 = n^2m is defined by
For all 0 \in B, n_1 \in I_0 then / For all n_1 \in B, 0 \in I_0 then
\quad 0 \cdot n_1 = 0 \cdot n_1
0 = 0 / 0 = 0
\quad n_1 = 0
\quad 0 = 0
\quad 0 \in I_0
\quad 0 \in I_0

For all n_2, 0 \in B, then, / For all n_1 \in B, 0 \in I_0 then,
\quad n_1 = n_1 \cdot 0
n_1 = n_1 \cdot 0
\quad 0 = 0
\quad 0 = 0
\quad 0 \in I_0
\quad 0 \in I_0

For all n_2 \in B, n_1 \in I_0 then / For all n_1 \in B, n_1 \in I_0 then,
\quad n_2 \cdot n_2 = n_2 \cdot n_1
n_2 \cdot n_2 = n_2 \cdot n_1
\quad n_1 = n_1
n_1 = n_1
\quad n_1 = n_1
n_1 = n_1
\quad n_1 = n_1
n_1 = n_1
\quad n_1 = n_1
n_1 = n_1
\quad n_1 = n_1
n_1 = n_1

Hence, I_0 satisfies all the requirements.
Therefore, we choose I_0 as I_j. Hence for I_1 = \{0, n_1, n_2\} and B = \{0, n_1, n_2, n_3\}

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\( n_i n_j^2 = n_j n_i \), for all \( n_i, n_j \in B \), and \( 0 \in I_1 \).

Therefore, \( I_1 \) also choose as \( I_j \).

Similarly, for \( I_2 = \{0, n_1, n_2\} \) and \( B = \{0, n_1, n_2, n_3\} \)
\( n_1 n_3^2 = n_3 n_1 \), for all \( n_i, n_j \in B \), and \( 0 \in I_2 \).

Therefore, \( I_2 \) also choose as \( I_j \).

Similarly, for \( I_3 = \{0, n_2, n_3\} \) and \( B = \{0, n_1, n_2, n_3\} \)
\( n_2 n_3^2 = n_3 n_2 \), for all \( n_i, n_j \in B \), and \( 0 \in I_3 \).

Therefore, \( I_3 \) becomes \( I_j \).

**Step 8 :** Verify that \( \cap I_j = I_0 \neq \{0\} \subseteq B \)
\( I_0 \cap I_1 \cap I_2 \cap I_3 = \{0, n_1\} \cap \{0, n_1, n_2\} \cap \{0, n_1, n_2, n_3\} \)
\( = \{0, n_1\} \neq \{0\} \subseteq B \)

**Step 9 :** If step (8) is true, then we write \( B \) is a Smarandache-Boolean-ring.

**REFERENCES :**


