ANALYSIS OF HAMMING WINDOW USING ADVANCE PEAK WINDOWING METHOD

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ABSTRACT

Many Window functions are widely used in digital signal processing for various applications in signal analysis and estimation, digital filter design and speech processing. In literature many windows have been proposed like ultra spherical window, Kaiser Window and hamming window with different specifications. But since they are suboptimal solutions, as there is a tradeoff between various factors and the best window depends upon the related application. The paper is based upon the performance of various windows. The Kaiser window has been used to design a better FIR Filter in terms of ripple ratio, side-lobe roll off ratio and main-lobe width with its advantages of no power series expansion in frequency domain hence need less hardware. The spectral characteristic of Hamming window is compared with other windows like Kaiser Window. A modification is also introduced in Hamming window to perform better in terms of ripple ratio as compared to Kaiser using advanced Peak windowing method.

Index Terms — FIR filter design, Hamming window, Kaiser Window, Main-lobe width, Side-lobe roll-off ratio, Ultra spherical window, window function.

I. INTRODUCTION

Many Window functions are widely used in digital signal processing for various applications in signal analysis and estimation, digital filter design and speech processing [1]. In literature many windows have been proposed like ultra spherical window, Kaiser Window and hamming window with different specifications. But since they are suboptimal solutions, as there is a tradeoff between various factors and the best window depends upon the related application.

Kaiser window(also called I₀-sinh window) [3] is a well known flexible window and widely used for the spectrum analysis and FIR filter design applications since it achieves a close approximation with the discrete prolate spheroidal functions that have the maximum energy concentration in the main-lobe. Because of the difficulty of computing the prolate function, a much simpler approximation using the zeroth-order modified Bessel function of the first kind was used which resulted in Kaiser Window, defined as power amplifier at the transmitter end. Clipping degrades the bit-error-rate (BER) performance and causes spectral spreading. One way to solve this problem is to force the amplifier to work in its linear region. Unfortunately, such a solution is not power efficient. Power efficiency is necessary in wireless communication as it provides adequate area coverage, saves power consumption, and allows small-size terminals. It is, therefore, important to aim at a power efficient.

FFT based measurements are subject to errors from an effect known as leakage. This effect occurs when the FFT is computed from of a block of data which is not periodic. To correct this problem appropriate windowing functions must be applied. The user must choose the appropriate window function for the specific application. When windowing is not applied correctly, then errors may be introduced in the FFT amplitude, frequency or overall shape of the spectrum. This application note describes the phenomenon of leakage, the various windowing functions and their strengths and weaknesses, and examples are given for various applications.

In this paper, we propose an advanced peak windowing method. The proposed method overcomes the drawback of the conventional one while maintaining almost the same spectral mask and providing more efficient BER performance. Through a numerical analysis and computer simulation, we show that the proposed scheme can be implemented by using a matrix form and exceeds the conventional windowing method.
II. SYSTEM MODEL

Fig. 1 shows an OFDM block diagram under consideration. The binary information bits are mapped to complex-valued MQAM symbols in a 2-dimensional signal constellation. The output of the mapper is serial-to-parallel converted and processed using an N-point complex inverse fast Fourier transform (IFFT). The N complex-valued time domain signals are then followed by a guard interval (GI), which contains the number of last $L-1$ samples ($N > L$). The GI consists of a partial repetition of an OFDM symbol so it does not affect the PAPR. Therefore, we do not take the GI into consideration here. Passing through a PAPR reduction block such as peak windowing, the signals undergoes a digital-to-analog conversion and are transmitted after high power amplifier.

At the receiver, the received signals can be demodulated by the reverse process of the transmitter.

III. ADVANCED PEAK WINDOWING METHOD

In this section, we outline the conventional peak windowing method and propose a new PAPR reduction technique. The proposed method overcomes the drawback of the conventional method when successive peaks emerge within a half of the window size.

The clipping method is the simplest way to reduce PAPR. However, it distorts signals nonlinearly and significantly increases the out-of-band radiation. A different approach is to multiply large signal peaks with a certain window function. In order to maintain the out-of-band radiation within a certain level, it is beneficial to increase the window length. On the other hand, the window should not be too long, because a long window length implies that many signal samples are affected, which degrades the BER performance.

Examples of suitable window functions are the Cosine, Kaiser, Hamming, and Hanning window [11]. In general, Kaiser Window is used because it is easy to shape spectrum by changing window length and shape parameter [12]. The Kaiser Window function with window length $M+1$ and shape parameter $\beta$ is given by

$$w(n) = 0.54 - 0.46 \cos \left( \frac{n \pi}{N} \right), \quad 0 \leq n \leq N \quad (1)$$

Where the window length is $L = N+1$, $\alpha$ is defined as $\alpha = M/2$, $I_0(x)$ represents the zeroth order modified Bessel function of the first kind, and $M$ is a positive even number.

The peak windowing can be expressed as a multiplication of input signals with a scale function at the peak point [7]. It can be accomplished by

$$x(n) = s(n)x(n), \quad (2)$$

Where, $s(n)$ means the scale function that is used to reduce the peak signal level. In addition, the scale function can be expressed as a convolution between weighting coefficient $c(n)$ and window function $w(n)$:

$$s(n) = 1 - \sum_{k=-\infty}^{\infty} c(k)w(n-k) \quad (3)$$

If we assume the input complex-valued data symbol of $N$ subcarriers as $X_k$ for $k = 0, L, N-1$, the output signal of the IFFT block is given by

$$x(t) = \sum_{k=0}^{N-1} X_k e^{j2\pi f_k t}, \quad 0 \leq t \leq NT,$$

where $f_k$ is the frequency of the $k$-th subcarrier defined as $f = k f_s (f = 1/NT)$ and $T$ is the sample interval.
In order to apply the scale function at the highest value among the oversampled signals exceeding the given threshold level, the peak sample index \( n_i \) and its value \( x(n_i) \) should be defined as

\[
|x(n_i)| = \max_{n_i, n_i} |x(n)|, \quad (4)
\]

where \( n_i \) is the non-uniformly spaced sample index running over the specific set of samples, which exceed the threshold \( A \). Also, \( n_i \) represents a sample index on the rising edge of the signal, where it first exceeds the threshold \( A \), while \( n_i \) represents a sample index, where the signal peak dips below the threshold \( A \). The weighting coefficient \( c(n_i) \) can be chosen in a way that the resulting envelope \( x_s(n) \) no longer crosses the desired threshold level \( A \) at the peak point:

\[
c(n_i) = 1 - A / |x(n_i)| \quad (5)
\]

Such a peak windowing method can limit the peak value to the threshold level while maintaining its spectrum. In a real system, however, when successive peaks occur within a half of the window size, windows will unfortunately overlap. As a result, the signal is suppressed much more than the required threshold and causes BER performance degradation. A solution for mitigating this effect is proposed in [7] by applying FIR filter with feedback structure. We refer to it as feedback-structured peak windowing (FPW) method. The basic idea of FPW is to scale down the weighting coefficients if necessary, by using the feedback structure. More details about FPW can be found in [7]. This idea, however, still has a limitation since it cannot avoid the signal degradation when successive peaks appear.

**IV. PROPOSED PEAK WINDOWING METHOD**

In the following we refer to it as advanced peak windowing (APW) method. The APW is aiming towards detecting the high instantaneous signal peaks and suppressing them to the exact threshold level even in case of consecutive peaks. More specifically, we generate new weighting coefficient \( \tilde{c}(n_i) \) instead of \( c(n_i) \) in order to avoid excessive suppression.

**V. COMPARISON OF WINDOW SPECTRUM**

In this section, we compare the performance of Hamming and Kiser windows with MATLAB.

**Hamming Window** - It has the shape of

\[
w(n) = \begin{cases} 
0.54 - 0.46 \cos \left( \frac{2\pi n}{M} \right), & 0 < n < M \\
0, & \text{otherwise}
\end{cases}
\]

Figure 3a shows Frequency response of Hamming window. Hamming window is one of the most simple window functions, and a member of the cosine-on-pedestal family. As can be seen, the main lobe of the Hamming window, like all general window functions is wider than the rectangular window, but that the side lobe levels are lower. This can be seen, since the application of a window involves convolving the ideal frequency response with the frequency of the window, the smaller the side lobes, the smaller the resulting ripples in the designed filter. The main lobe width of a window is related to the transition width of the designed filter. The trick is to select the window type and filter length that will give a filter with...
the correct rate of roll-off and level of attenuation in the stop band.

**Kaiser Window** - The Kaiser window has the following shape:

\[
 w(k) = \begin{cases} 
 I_0[\beta(1-\left[n-\frac{M}{2}\right]{\left(M/2\right)}^{2.5})], & 0 < n < M \\
 0, & \text{otherwise} 
\end{cases}
\]

Where \( \beta \) is the tuning parameter of the window to it shows a trade-off between the desired “side lobe peak-main lobe width,” and \( I_0(.) \) is the zero order modified Bessel function of the first kind. From simulated result, shown in fig 3b it is observed that for \( M=200 \) and \( \beta =2.5 \), the window have side lobe peak (~ -21.1 dB). The Kaiser family of window functions has received particular attention due to the near-optimal tradeoff between main lobe width and side lobe area. Table 1 shows the comparison between the Hamming and Kaiser Window.

<table>
<thead>
<tr>
<th>Window</th>
<th>Leakage factor (%)</th>
<th>Side Lobe attenuation Factor (db)</th>
<th>Main Lobe Width (-3db)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hamming</td>
<td>0.04</td>
<td>41.8</td>
<td>0.0781 25</td>
</tr>
<tr>
<td>Kaiser</td>
<td>1.32</td>
<td>21.1</td>
<td>0.0097 656</td>
</tr>
</tbody>
</table>

The comparison of Hamming and Kaiser Window in terms of the ripple ratio suggests that Kaiser Window gives lesser ripple ratio than hamming window, and the difference becomes larger as the normalized width increases.

On the other hand it is observed that hamming window gives better roll-off ratio than Kaiser, and the difference becomes larger as the normalized width increases.

By comparing Kaiser and the combinational windows including Hamming window for a fixed window length and main-lobe width, it can be observed that the maximum side-lobe amplitude for the ripple ratio occurs in the first side-lobe but that it lies in the third side-lobe for the combination of Kaiser and Hamming windows.

**VI. SIMULATION RESULTS**

In the simulation, we assume that the OFDM system consists of 1024 subcarriers with 16-QAM. We also assume that the oversampling factor is four. We also assume that the guard interval is one by four. If we oversample the OFDM signal by a factor of four, the PAPR of the discrete signal is almost the same as that of continuous signal [9].

![Comparison of peak suppression to the given threshold by previous and proposed method.](image)
Fig. 4 depicts the amplitude difference between the FPW and the APW in Kaiser and hamming. This difference eventually affects the BER performance at the receiver. Nevertheless, we can clearly see that the amplitude of the proposed method is much closer to the given threshold level.

The baseband signal power spectral density is compared in Fig. 5. The Hamming method shows several regrowth of the Side-lobe characteristic. However, the Hamming and the Kaiser have lot of differences on the same spectrum characteristics. From this figure we can find that for Hamming window technique at normalized frequency -1 and 0.999 MHz the spectral density is -81.5 and -46.05 respectively. Similarly for Kaiser Window technique at normalized frequency -2 and 1.998 the spectral density is -81.5 and -46.05. So the power spectral density is same for different normalized frequency but they are equal for maximum and minimum values.

VII. CONCLUSION

This paper proposes an advanced peak windowing method, which is referred to as APW, effectively suppress the peak signals to the desired threshold level in case those successive peaks occur within a half of the window length. Through intensive computer simulations, we have seen that difference between the performance improvements can be achieved by the Hamming window over the Kaiser Window spectral characteristics. But Kaiser Window gives better ripple ratio as compared to Kaiser Window. This drawback can be overcome by the combination of hamming window with cosh window. Further the complexity can be reduced by introducing a modified window having a new adjusting parameter similar to that also increases the main lobe width and reduces the side lobe ratio and ripple ratio. Width. But Kaiser Window gives better ripple ratio as compared to Kaiser Window. This drawback can be overcome by the combination of hamming window with cosh window. Further the complexity can be reduced by introducing a modified window having a new adjusting parameter similar to that also increases the main lobe width and reduces the side lobe ratio and ripple ratio.

VIII. REFERENCES


Bibliography


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